ANALYSIS II Homework 6 27.04.2016

1. Evaluate the following integrals by using definition of definite integral.

(a) $\int_0^b \cos x dx = ?$ **(b)** $\int_0^b \sin x dx = ?$

2. Show that $\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{n^2 - k^2} = \int_0^1 \sqrt{1-x^2}\, dx$.

3. Prove the following:

(a) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n+k+1} = \ln 2$ **(b)** $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{n^2} = \frac{1}{2}$ **(c)** $\lim_{n\to\infty} \frac{1}{n^{p+1}} \sum_{k=1}^{n} k^p = \frac{1}{1+p}$

4. Evaluate the following limits:

(a) $\lim_{x\to 0} \frac{\int_0^x \sin\left(t^3\right) dt}{x^4} = ?$ **(b)** $\lim_{x\to 0} \frac{\sin 2x - 2\int_0^x e^{\sin t} dt}{\arctan\left(x^2\right)} = ?$

(c) $\lim_{x\to 0} \frac{e^x - 1 + \int_x^0 e^{\arctan t} dt}{1-\cos x} = ?$ **(d)** $\lim_{x\to 0} \frac{x}{1-e^{(x^2)}} \int_x^0 e^{(t^2)} dt = ?$

5. Evaluate the following definite integrals:

(a) $\int_0^{\pi/2} \frac{dx}{a^2 - b^2 \sin^2 x} = ? \ (a > b) \left(Answer. \frac{\pi}{2a\sqrt{a^2 - b^2}} \right)$

(b) $\int_0^{\pi/2} \frac{\sqrt{\sin x}\, dx}{\sqrt{\sin x} + \sqrt{\cos x}} = ? \ \left(Answer. \frac{\pi}{4} \right)$ **(c)** $\int_e^{e^2} \frac{dx}{x \ln^3 x} = ? \ \left(Answer. \frac{3}{8} \right)$

(d) $\int_0^{1/\sqrt{2}} \frac{x \arcsin\left(x^2\right) dx}{\sqrt{1-x^4}} = ? \left(Answer. \frac{\pi^2}{144} \right)$

6. Suppose that $a>0$ and that f is integrable on $[-a,a]$.

(a) If f is a even function on $[-a,a]$, show that $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

(b) If f is a odd function on $[-a,a]$, show that $\int_{-a}^{a} f(x)\,dx = 0$

7. If f is continuous function on $[-a,a]$, show that $\int_{-a}^{a} f\left(x^2\right)dx = 2\int_{0}^{a} f\left(x^2\right)dx$

8. Find $\frac{dF}{dx}$ if **(a)** $F(x)=\int_{0}^{x^2}\frac{1}{1+t^3}\,dt$ **(b)** $F(x)=\int_{x^2}^{x}\sqrt{1+t^2}\,dt$

9. Let $f:[0,3]\to\mathbb{R}$ be defined by $f(x)=\begin{cases} x\,, & if\ 0\le x<1 \\ 1\,, & if\ \le x<2 \\ x\,, & if\ 2\le x\le 3 \end{cases}$.

Obtain the formula $F(x)=\int_{0}^{x} f(t)\,dt$ and skecth the graph of f and F.

Where is F differentiable? Evaluate $F'(x)$ at all such points.

10. The function g is defined on $[0,3]$ by $g(x)=\begin{cases} -1\,, & if\ 0\le x<2 \\ 1\,, & if\ 2\le x\le 3 \end{cases}$.

Find the indefinite integral $G(x)=\int_{0}^{x} g(t)\,dt$ for $0\le x\le 3$ and skecth the graph of g and G.

Does $G'(x)=g(x)$ for all x in $[0,3]$.

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