ANALYSIS I Homework 3 11.12.2015

1. The sequence (a_n) is given. If $a_{2n} \to a \in \mathbb{R}$ and $a_{2n} - a_{2n-1} \to 0$,

then show that $a_n \to a$ . .

2. The sequences (a_n) ve (b_n) are given. Let $L \in \mathbb{R} - \{0\}$ such that $a_n \to L$.

If $\lim\limits_{n\to\infty}(a_n b_n)$ exits, then Show that $\lim\limits_{n\to\infty}(b_n)$ exists.

3. Prove the following by using definition of limits for function.:

(a) $\lim\limits_{x\to 2}\left(2x - x^2\right) = 0$ **(b)** $\lim\limits_{x\to -2}\dfrac{x^2}{x+6} = 1$ **(c)** $\lim\limits_{x\to 1}|2x-1| = 1$

4. Find two functions f, g defined on $\mathbb{R}$ such that $0 = \lim\limits_{x\to 0}\left(f(x) + g(x)\right) \neq \lim\limits_{x\to 0} f(x) + \lim\limits_{x\to 0} g(x)$

5. Let $f : A \to \mathbb{R}$ be a function and $\forall x \in A$ $f(x) > 0$. If $\lim\limits_{x\to a}\left(f(x)\right)^2 = L \in \mathbb{R}$, Then:

is $\lim\limits_{x\to a^2} f(x) = L$ or $\lim\limits_{x\to a^2} f(x) \neq L$? Explain your answer.

5. Let $f : X \to Y$ be a function and $A \subseteq X$, $B \subseteq X$, $C \subseteq Y$. Then Prove the follwings:

(a) f is $(1-1)$ $\Leftrightarrow$ $f(A \cap B) = f(A) \cap f(B)$ for all A and B

(b) Give an example such that $f(A \cap B) \neq f(A) \cap f(B)$

(c) f is onto $\Leftrightarrow$ $f^{-1}(C) \neq \varnothing$ for all $C \neq \varnothing$. and B is (1-1)

(d) f is onto $\Leftrightarrow$ $f\left(f^{-1}(C)\right) = C$

(e) If f is $(1-1)$ and onto, then $f(C_X A) = C_Y f(A)$

(f) f is $(1-1)$ $\Leftrightarrow$ then $f^{-1}\left(f(A)\right) = A$

6. Prove that the following limits do not exist.

(a) $\lim\limits_{x\to -\pi} Sin\left(\dfrac{\pi}{x+\pi}\right)$ **(b)** $\lim\limits_{x\to \pi} Cos\left(\dfrac{\pi}{x-\pi}\right)$ **(c)** $\lim\limits_{x\to +\infty} Cos(\pi x)$

7. Find the limits:

(a) $\lim\limits_{x\to\pi}(x-\pi)\tan\left(\dfrac{x}{2}\right)$ **(b)** $\lim\limits_{x\to 0}\dfrac{x+1}{\sqrt{6x^2+3}+3x}$ **(c)** $\lim\limits_{x\to\pi}\dfrac{1+Cos(x)}{(x-\pi)^2}$

(d) $\lim\limits_{x\to +\infty}\left(\dfrac{2+x}{2-x}\right)^{4-x}$ **(e)** $\lim\limits_{x\to \pi/4}\dfrac{\sin x - \cos x}{\tan x - 1}$ **(f)** $\lim\limits_{x\to 0}\dfrac{1-\sqrt{\cos x}}{1-\cos\sqrt{x}}$