

Pharmacokinetic Calculations

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- Logarithm is a mathematical function that is the inverse of exponential functions.

$$x = b^y \quad \log_b(x) = y$$

If the base (b) is 10, the function is called **decimal logarithm**
or
general logarithm.

If the base (**b**) is «**e**», then it is described as **natural logarithm (Ln)**
e= 2,7182

$$x = b^y \quad \log_b(x) = y$$

$$\log_2 8 = 3$$

$$\log_2 2^3 = 3$$

$$\log_{10} 1000 = 3$$

$$\log_{10} 10^3 = 3$$

$$\log 3 = \log_{10} 3$$

Logarithmic Calculations

$$\log_b (x.y)=\log_b x + \log_b y$$

$$\begin{aligned}\log_2 (32.8) &= \log_2 32 + \log_2 8 \\ &= \log_2 2^5 + \log_2 2^3 \\ &= 5 + 3 \\ &= 8\end{aligned}$$

Logarithmic Calculations

$$\log_b (x/y) = \log_b x - \log_b y$$

$$\log_2 (64/4) = \log_2 64 - \log_2 4$$

$$= \log_2 2^6 - \log_2 2^2$$

$$= 6 - 2$$

$$= 4$$

$$\log_2 16$$

Logarithmic Calculations

$$\log_b (x^p) = p \log_b x$$

$$\log_2 (2^6) = 6 \log_2 2 = 6$$

$$\text{Log } 1000 = 3$$

$$\text{Ln } 1000 = 6,9077$$

$$2,7182(e)^{6,9077} = 1000$$



$$\text{ln } N / \text{log } N = 2,303$$

Anti-logarithm

- It is the opposite function of the logarithm.

Anti-logarithm of 0,028

$$10^{0,028} = 1,0666$$

To calculate the inverse of the natural logarithm:

$$e^{-1,3} = 0,2725$$

Differential Equations

- Equations involving derivatives of one or more functions are called differential equations.
- The description of most scientific problems involves the variation of some key variables relative to other variables.

- By expressing the rate of change of variables with derivatives, differential equations are obtained which provide definite (exact) mathematical formulations for physical principles and laws.
- Differential equations express a physical model and are called as mathematical model.

- **The derivative** is the amount of change at any time interval. It is used to measure “**the change**”.
- **The integral** is used to express the total change in a given range, or the cumulative amount of range.

“The derivative is expressed by the slope of the tangent line drawn to a certain point on the graph.”

*The more perpendicular the tangent is,
the change is the faster.*

*The more horizontal the tangent is,
the change is the slower.*

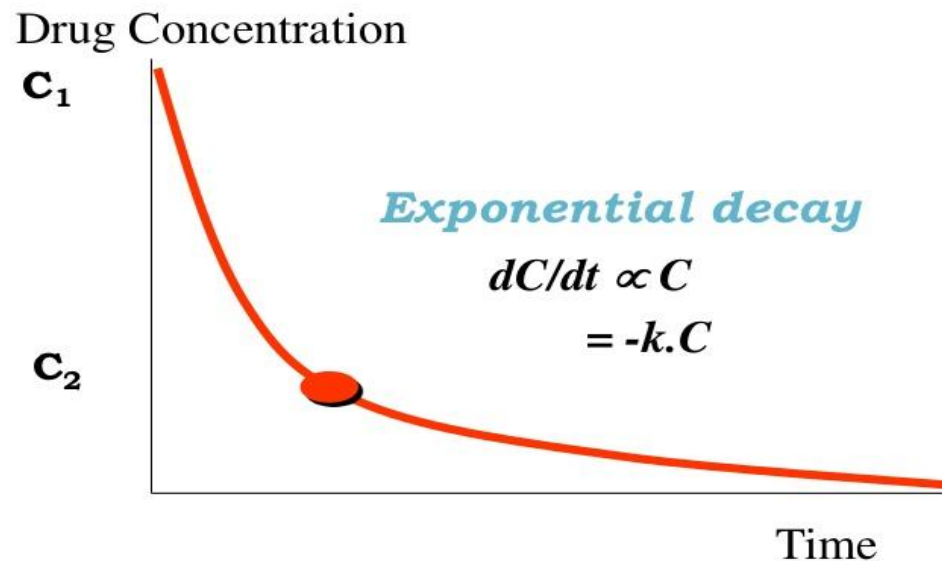
The amount of the active substance in the organism varies with time.

The quantity; is a dependent variable, the time is an independent variable.

The amount which is eliminated at a time of unit can be described by **an elimination rate**.

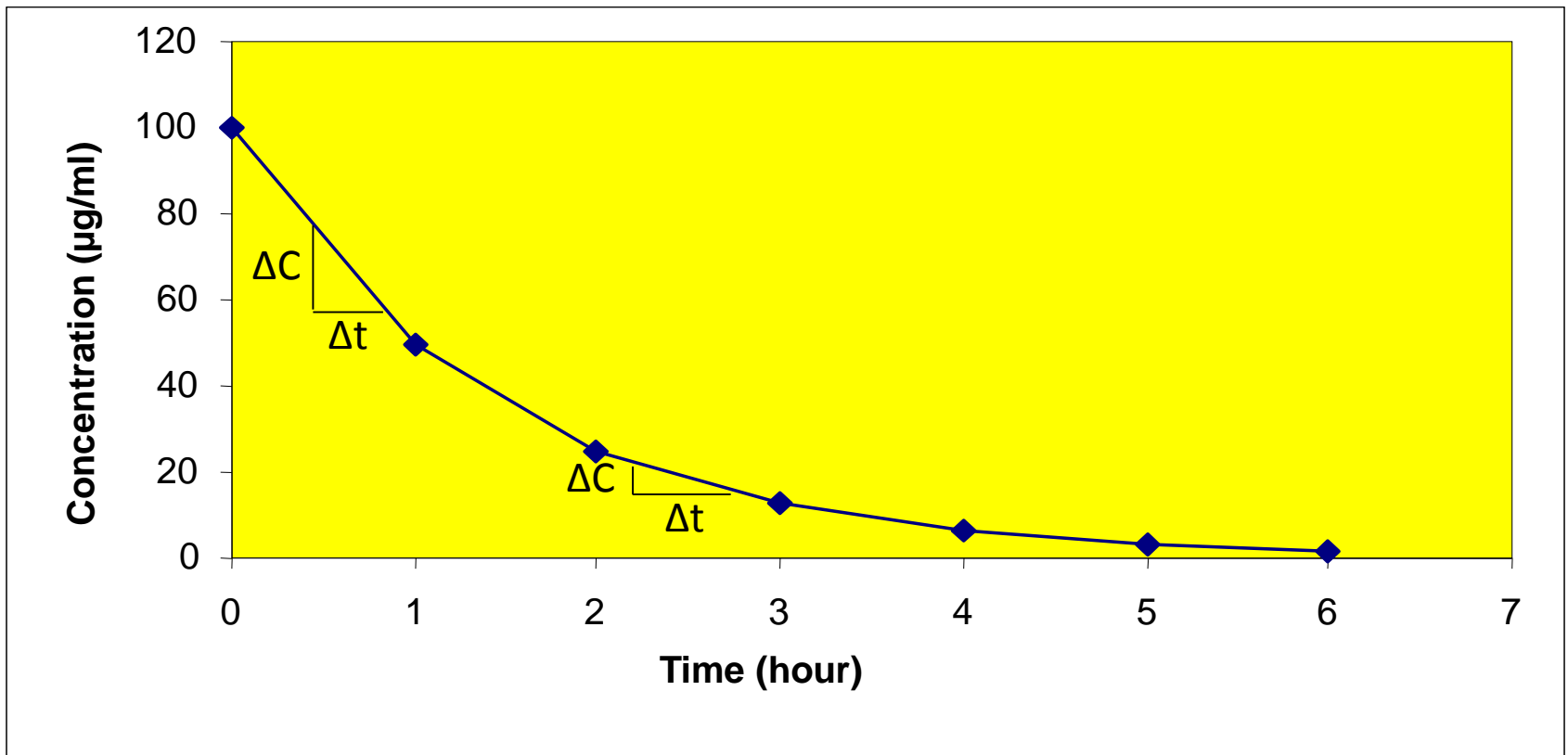
The calculation of the speed is done by differential functions.

Here, the purpose of the differential calculation is to determine the rate of change.



Determining the rate of change of a variable

Rate of concentration change dc/dt



The Slope of the Curve

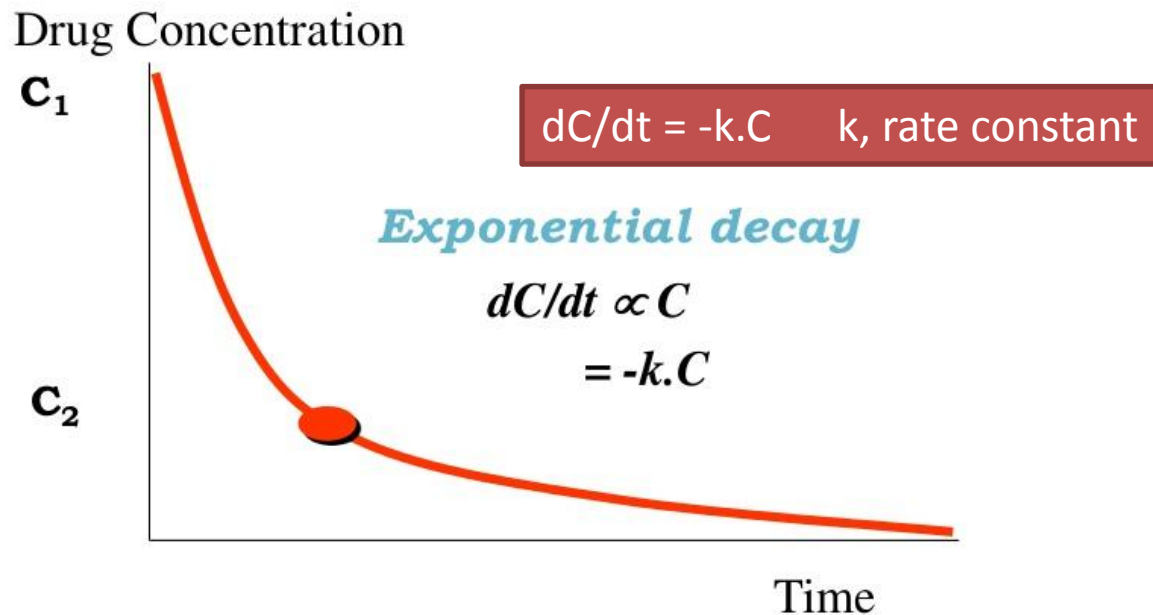
Time (hour)	Plasma drug const. (mcg/mL)
0	12
1	10
2	8
3	6
4	4
5	2

$$(10-8)/(2-1)$$

$$dc/dt = 2 \text{ mcg/mL/hour}$$

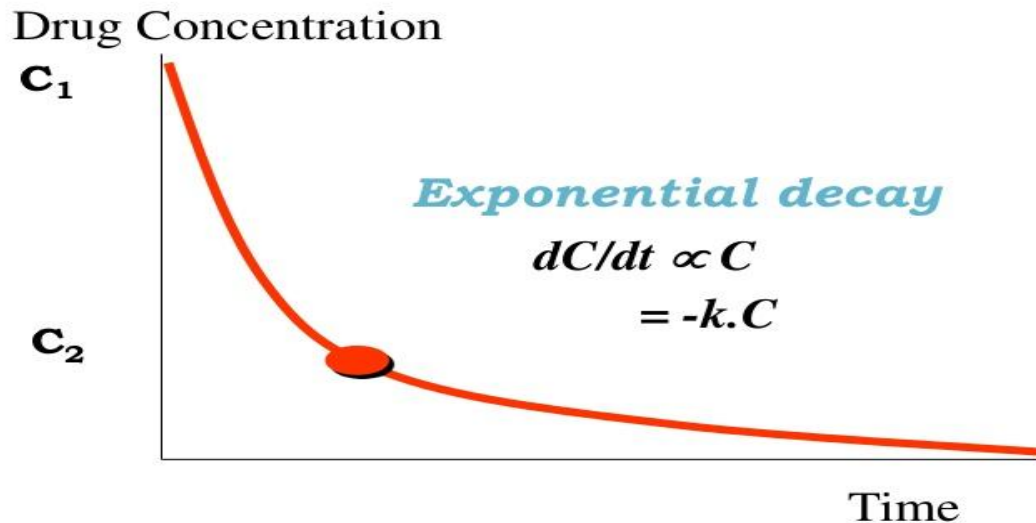
The integral function is based on integrating the differential equation.
Integral is obtained from the differential equation, which contains derivative, by using integration.

If the equation is integrated,
 $C = C_0 \cdot e^{-kt}$ equation is obtained.



Determining the rate of change of a variable

- Here, it is seen that the decrease in the amount or concentration is faster at the beginning and slower with time.
- This reduction is an exponential decrease.
- C_0 , is the drug concentration at the beginning (at $t=0$).



- $C = C_0 \cdot e^{-kt}$
- If the **natural logarithms** of both sides are taken:

$$\ln C = \ln C_0 - k_{el} \cdot t$$

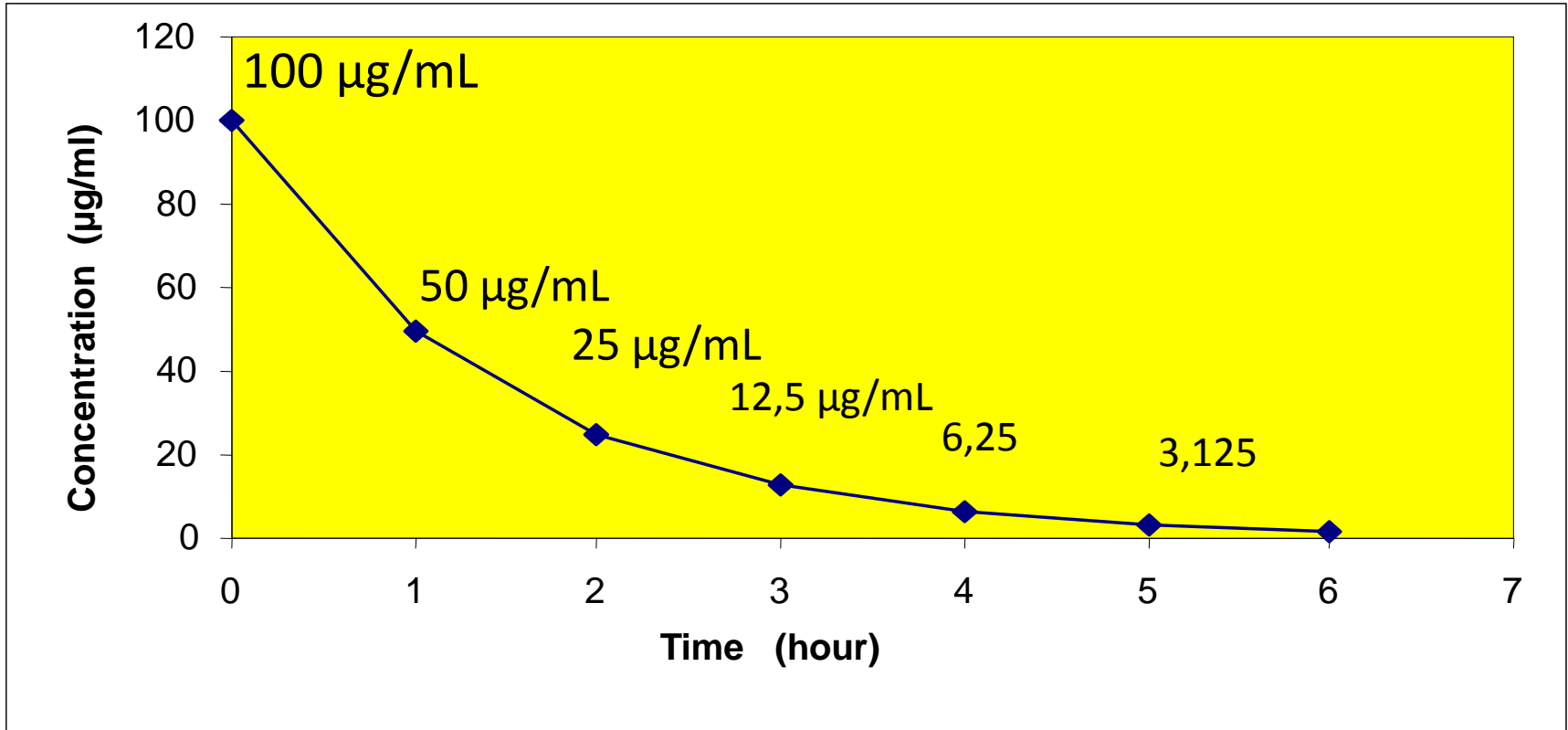
- $C=C_0 \cdot e^{-kt}$ equation can also be expressed logarithmically.
- If the **logarithms** of both sides are taken :

$$\text{Log } C = \text{Log } C_0 - (kt/2.303)$$

Example

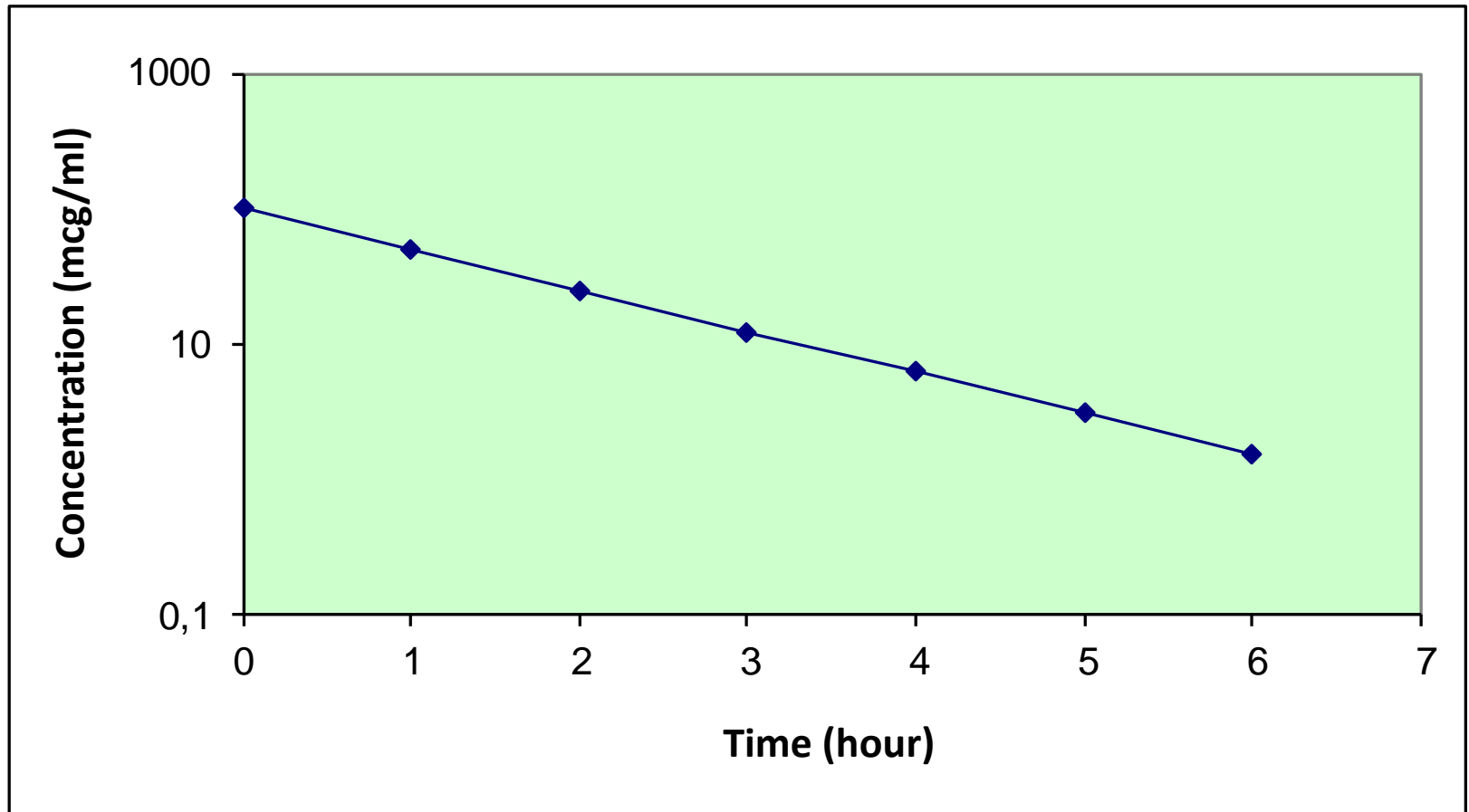
<u>t (hour)</u>	<u>C ($\mu\text{g}/\text{mL}$)</u>
0	100
1	50
2	25
3	12,5
4	6,25
5	3,125

Linear Graph



$C_0 = 100 \mu\text{g/mL}$

Semi logarithmic graph

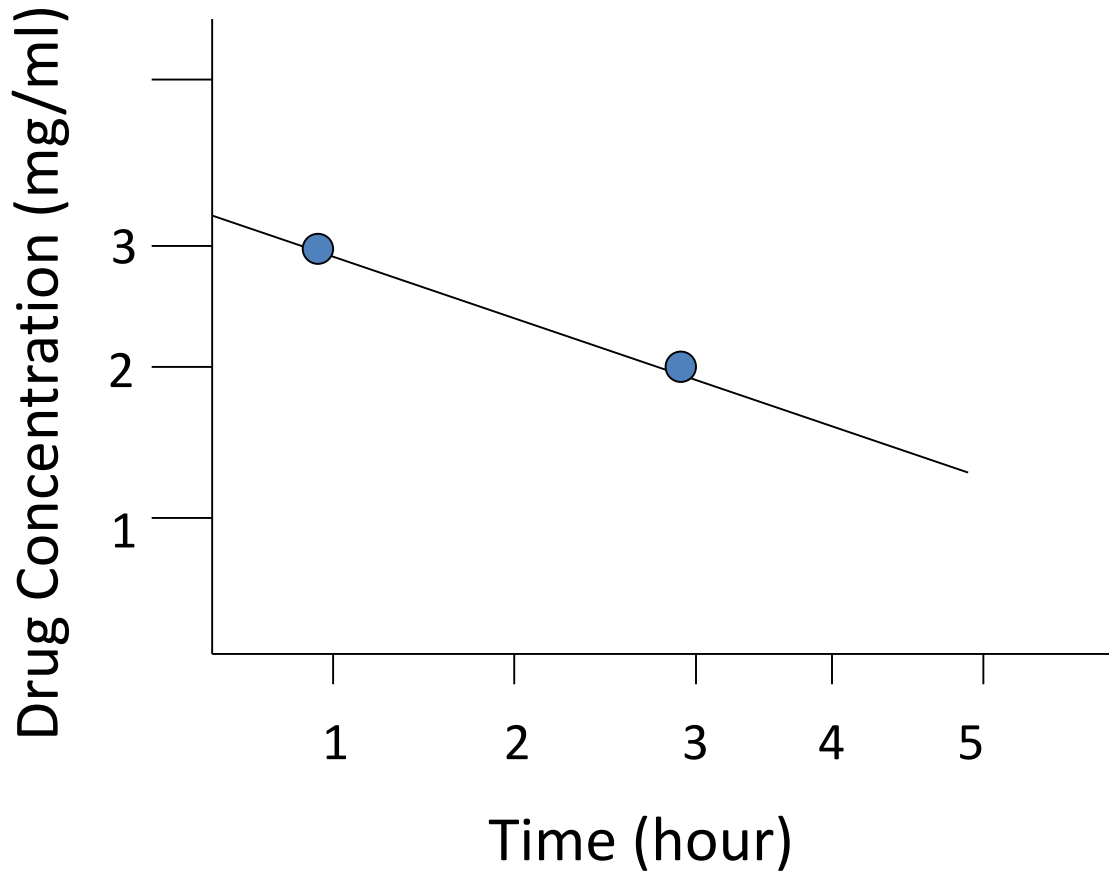


The important point is,
The values are marked without taking the logarithm.

Calculation of the elimination rate constant (k_{el})

- $\ln C = \ln C_0 - k_{el} \cdot t$ equation gives a curve; **the slope** of the curve gives the **elimination rate constant (k_{el})** verirr.
- k_{el} , can be found by a calculator or computer using In-linear regression function in Microsoft Office Excel.

Slope Linear



$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

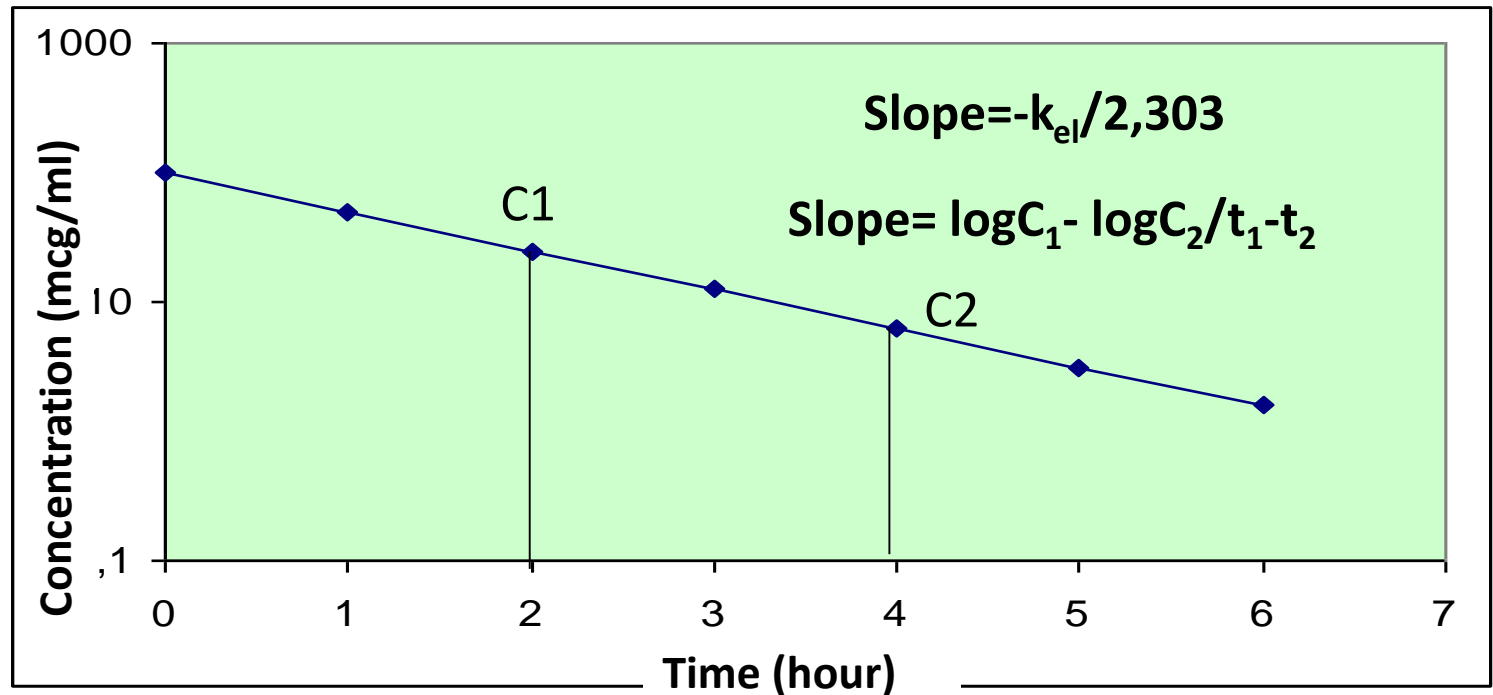
$$\text{Slope} = \frac{2 - 3}{3 - 1}$$

$$\text{Slope} = -\frac{1}{2}$$

$$y = -\frac{1}{2}x + 3,5$$

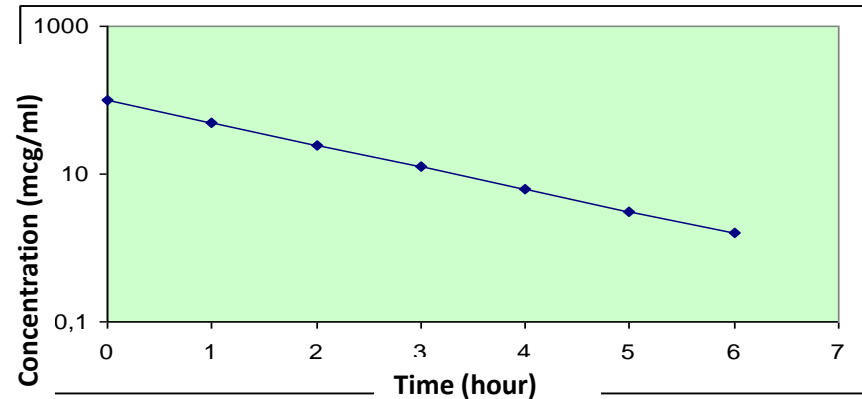
Calculation of the elimination rate constant (k_{el})

- The slope with log-linear regression = $-k_{el}/2,303$.
- In the ln-linear regression, the slope of the line directly gives the " k_{el} " value.



<u>t (hour)</u>	<u>C (µg/mL)</u>
0	100
1	50
2	25
3	12,5
4	6,25
5	3,125

$$\text{Log } C = \text{Log } C_0 - kt/2.303$$



$$\text{Log}C - \text{Log}C_0 = -k.t/2.303$$

$$\text{Log}C_0 - \text{Log}C = k.t/2.303$$

$$\text{Log}50 - \text{Log}25 = k. (1-2)/2.303$$

$$1,6989 - 1,3979 = -k/2.303$$

$$0,301 \times 2,303 = -k$$

$$k = - 0, 693$$

Calculation of the Elimination Half-Life

- The elimination half-life ($t_{1/2}$) of the drug in the plasma can be calculated from the following formula:

$$t_{1/2} = \ln 2 / k_{el} = 0,693 / k_{el}$$

- The half-life ($t_{1/2}$) can also be calculated using the graph.

Calculation of the Elimination Half-Life

- $\ln C = \ln C_0 - k_{el} \cdot t$

The time until C_0 reach to $C_0/2$ is called as $t_{1/2}$.

- In the line equation, replace t with $t_{1/2}$ and C with $C_0/2$,
 $\ln C_0/2 = \ln C_0 - k_{el} \cdot t_{1/2}$ or

$$\ln C_0/2C_0 = -k_{el} \cdot t_{1/2} \quad \text{or} \quad \ln 2 = -k_{el} \cdot t_{1/2}$$

$$= t_{1/2} = \ln 2 / k_{el}$$

I.V. SINGLE DOSE ADMINISTRATION SINGLE-COMPARTMENT MODEL

- **QUESTION 1:** Drug “A” is administered to a patient intravenously and blood was collected at certain time points. The following plasma concentrations were determined in these blood samples.

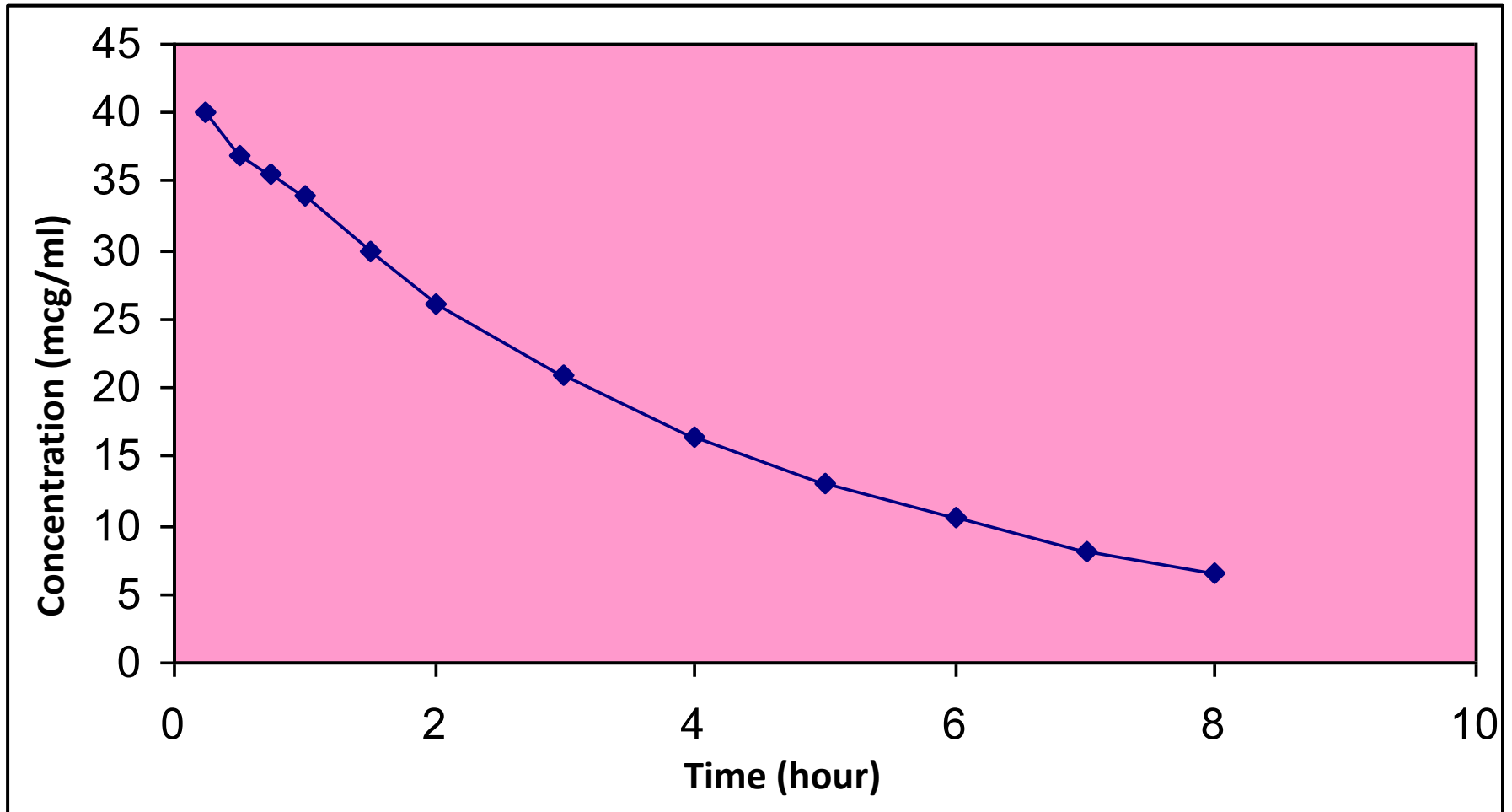
Plasma Data

t (hour)	0	0.25	0.5	0.75	1	1.5	2	3	4	5	6	7	8
C (μg /ml)	0	40	37	35,5	34	30	26	21	16,5	13	10,5	8	6,5

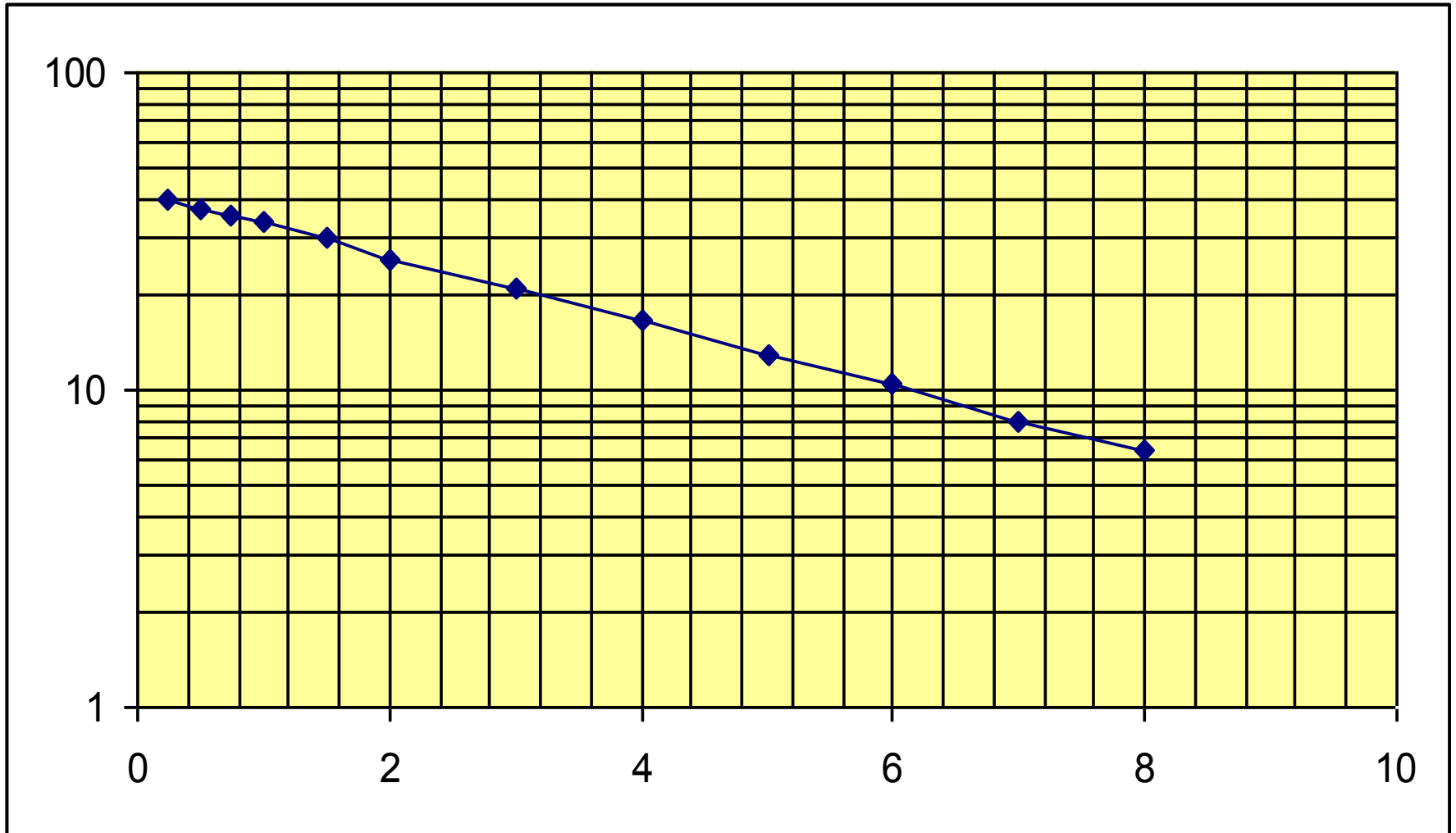
Questions

- Draw the graphs with the values in the table on normal paper and semi-logarithmic paper.
- Calculate the value " C_0 ".
- Calculate the elimination rate constant (k_{el}).
- Calculate the elimination half-life ($t_{1/2}$).
- Calculate the concentration at the time 3,5. hour.

Plasma const.-time curve



Semi logarithmic paper presentation



Calculation of C_0

- For the calculation of C_0 value, do In-linear regression (LR) by using the last 4 points.

<u>t (hour)</u>	<u>C ($\mu\text{g/ml}$)</u>
5	13
6	10,5
7	8
8	6,5

- Find the point where the Y axis is cut.
- k_{el} value is equal to the slope of the curve.
It is calculated as $k_{el} = -0,24 \text{ hour}^{-1}$.

$$y = 42,3 \cdot e^{-0,24x}$$

Calculation of k_{el} value

t	Const.	Log const.
5	13	1.113943
6	10.5	1.021189
7	8	0.90309
8	6.5	0.812913

$$\text{Slope} = \frac{\log c_1 - \log c_2}{t_1 - t_2}$$

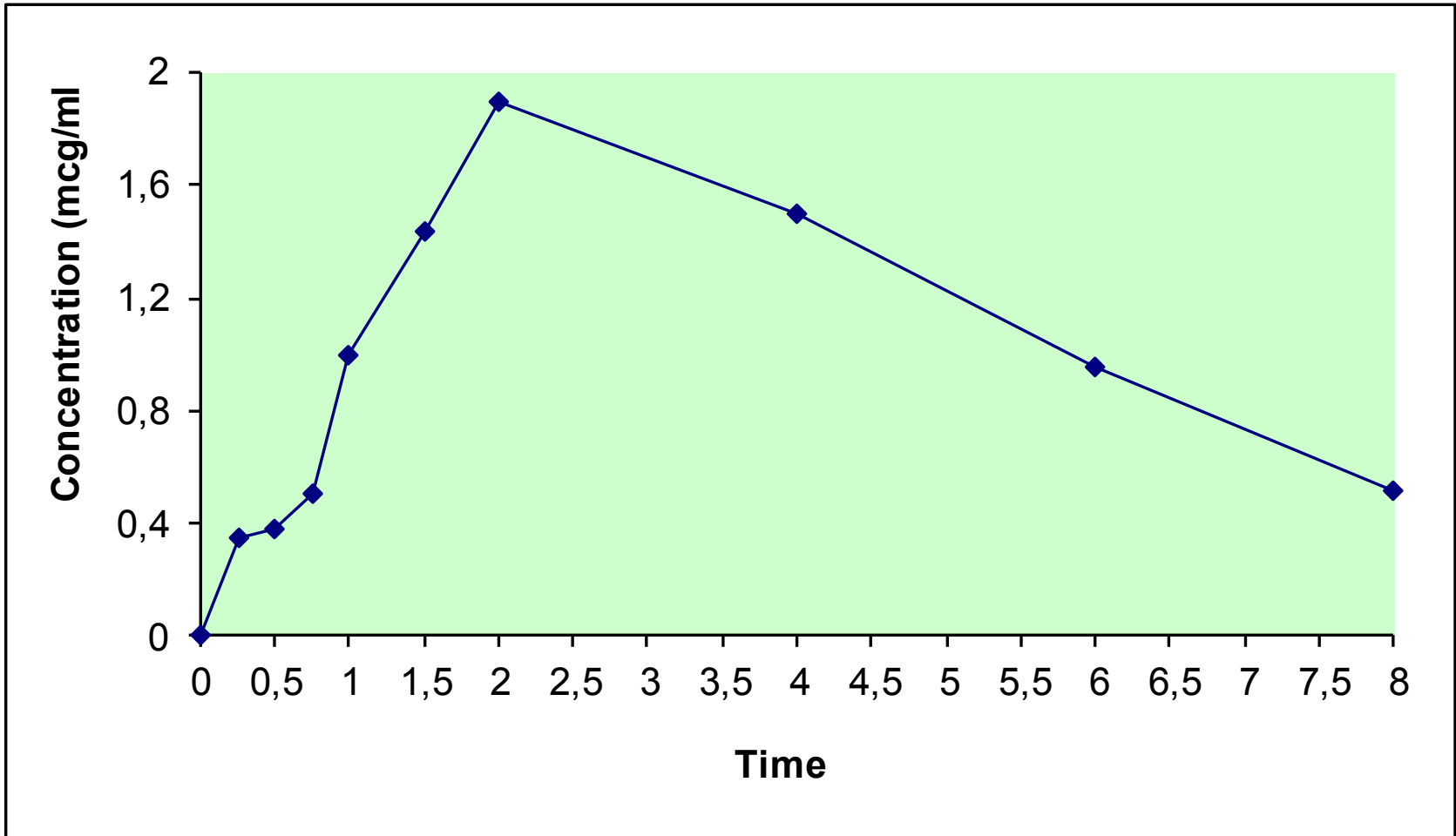
$$\text{Slope} = \frac{1.02 - 0.81}{6 - 8}$$

$$\text{Slope} = -0.104$$

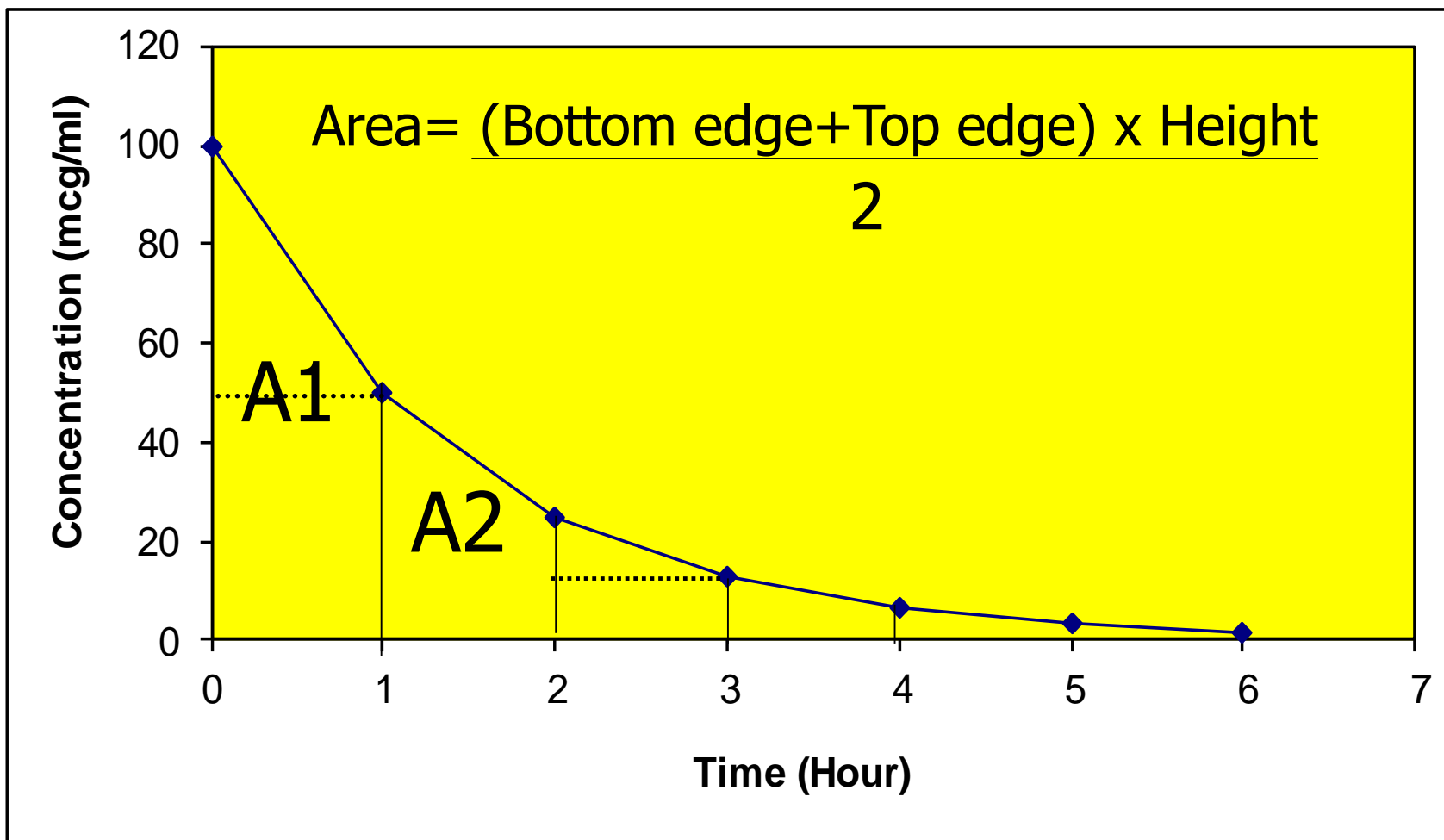
$$\text{Slope} = \frac{k_{el}}{2.303}$$

$$k_{el} = -0.24$$

Calculate the AUC values for the intervals of 0-8 ve 0- ∞ .



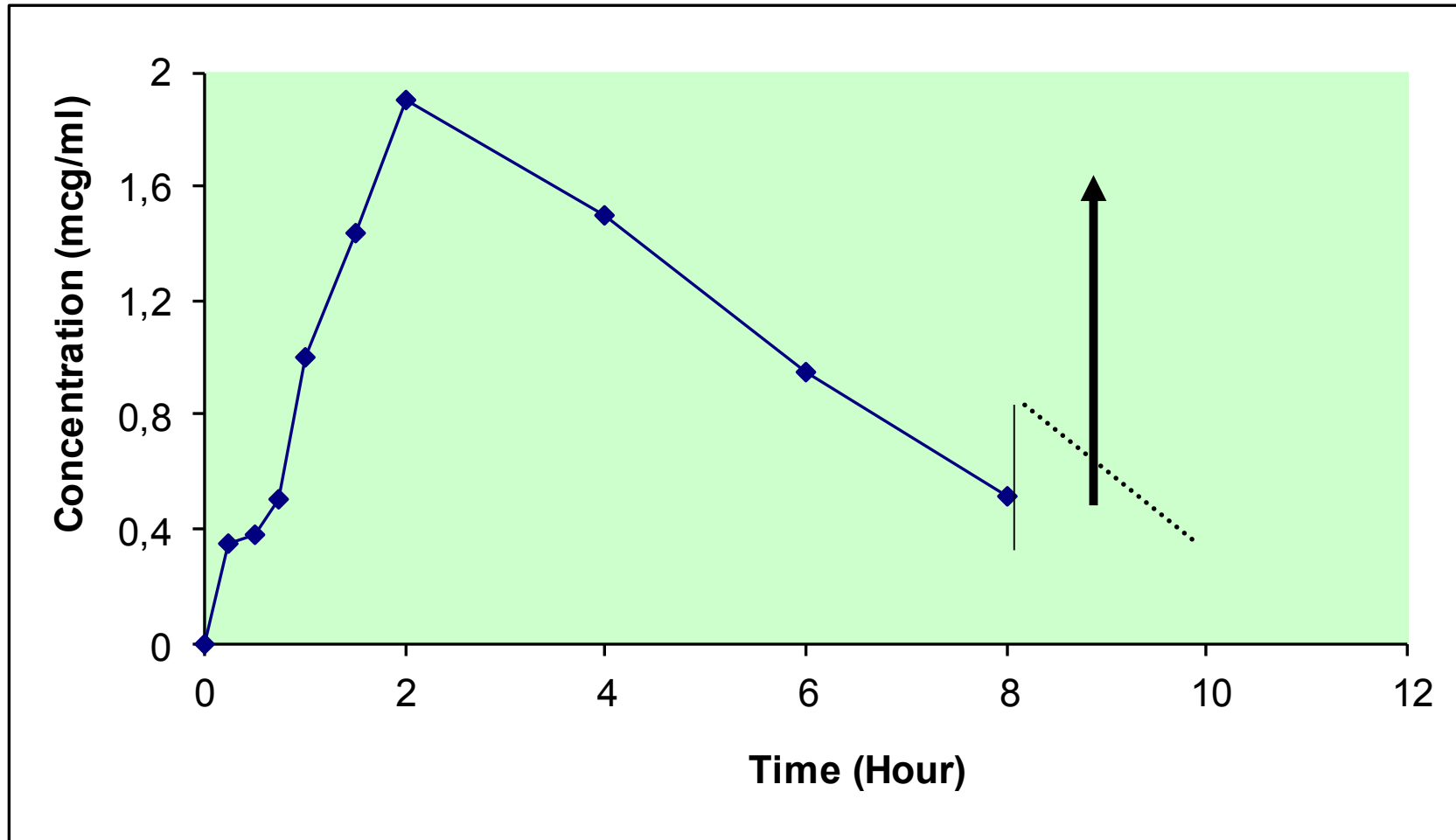
Calculation of AUC with Trapezoid Method



Calculation of AUC

- $AUC_1 = 0,0437 \mu\text{g}\cdot\text{h}/\text{ml}$
- $AUC_2 = 0,090 \mu\text{g}\cdot\text{h}/\text{ml}$
- $AUC_3 = 0,109 \mu\text{g}\cdot\text{h}/\text{ml}$
- $AUC_4 = 0,187 \mu\text{g}\cdot\text{h}/\text{ml}$
- $AUC_5 = 0,61 \mu\text{g}\cdot\text{h}/\text{ml}$
- $AUC_6 = 0,835 \mu\text{g}\cdot\text{h}/\text{ml}$
- $AUC_7 = 3,40 \mu\text{g}\cdot\text{h}/\text{ml}$
- $AUC_8 = 2,45 \mu\text{g}\cdot\text{h}/\text{ml}$

$$AUC_{8-\infty} = C_8 / k_{el}$$



Calculation of AUC

- $AUC_9 = 1,46 \mu\text{g}\cdot\text{h}/\text{ml}$
- $AUC_{0-8\text{h}} = 9,184 \mu\text{g}\cdot\text{h}/\text{ml}$

- $AUC_{t-\infty} = C_t / k_{el}$
 $AUC_{8-\infty} = C_8 / k_{el}$

$$= 0,51 / 0,266 = 1,96 \mu\text{g}\cdot\text{h}/\text{ml}$$

- $AUC_{\text{total}} = 9,184 + 1,96 = 11,14 \mu\text{g}\cdot\text{h}/\text{ml}$

ORAL SINGLE DOSE ADMINISTRATION SINGLE-COMPARTMENT MODEL

HOMWORK QUESTION

- **SORU-2** : Drug "A" is administered to a patient via oral route and blood was collected at certain time points. The following plasma concentrations were determined in these blood samples:

t (hour)	0	0,25	0.5	0.75	1	1,5	2	4	6	8
C (µg /ml)	0	0,35	0.375	0.5	1	1,44	1,9	1,5	0,95	0,51

Homework

- Draw the plasma concentration-time curve as Linear (without using Excel) and calculate the Area Under the Curve (AUC).
- Draw the plasma concentration-time curve exponentially using the Excel program for the concentrations corresponding to the last 4 time points (2., 4., 6., and 8. hours). Add the output to your homework.
- Calculate the elimination rate constant (k_{el}) for the last 4 time points by using Log and compare it with the slope of the curve which you obtained from the Excel program.
- What is the elimination half-life ($t_{1/2}$) of the drug?

Do not forget to write your name, surname and number to your homework.
Give your homework in a transparent file.