6.1. Hidrojen atomunun d durumundaki bir e^- için Hamiltonyen;

$$H = A + B\,\vec{L}\cdot\vec{S} + C\,\vec{L}\cdot\vec{L}$$

olarak veriliyor. A, B ve C sabitler. Toplam açısal momentum durumlarını ve bu durumlara tekabül eden özdeğerleri bulunuz.

	s	p	d	f	g
$l =$	0	1	$\boxed{2}$	3	4

$l = 2 \to m_l = -2, -1, 0, 1, 2$

$s = 1/2 \to m_s = -1/2, +1/2$

$$\vec{J} = \vec{L} + \vec{S} \Rightarrow j = |\vec{L} - \vec{S}|, \ldots, (L+S)$$

$$j = 5/2,\ 3/2$$

$$|l, s; j, m_j\rangle = \left.\begin{matrix} |2, 1/2, 5/2, m_j\rangle \\ |2, 1/2, 3/2, m_j\rangle \end{matrix}\right\}$$ özdurumlar (j'ye göre)

- $\vec{J} = \vec{L} + \vec{S} \Rightarrow J^2 = (\vec{L}+\vec{S})\cdot(\vec{L}+\vec{S}) = L^2 + S^2 + 2\vec{L}\cdot\vec{S}$

$$\vec{L}\cdot\vec{S} = \frac{J^2 - L^2 - S^2}{2}$$

$$\Rightarrow H = A + \frac{B}{2}(J^2 - L^2 - S^2) + CL^2$$

- $|2, 1/2, 3/2, m_j\rangle \Rightarrow$

$$H|2, 1/2; 3/2, m_j\rangle = \left(A + \frac{B}{2}(J^2 - L^2 - S^2) + CL^2\right)|2, 1/2; \tfrac{3}{2}, m_j\rangle$$

$$J^2|2, 1/2; 3/2, m_j\rangle = \hbar^2 \frac{3}{2}\left(\frac{3}{2}+1\right)|2, 1/2; 3/2, m_j\rangle$$
$$= \frac{15\hbar^2}{4}|2, 1/2; 3/2, m_j\rangle$$

$$L^2|2, 1/2; \tfrac{3}{2}, m_j\rangle = \hbar^2 2(2+1)|2, 1/2; 3/2, m_j\rangle$$
$$= 6\hbar^2|2, 1/2; 3/2, m_j\rangle$$

$$S^2|2, 1/2; \tfrac{3}{2}, m_j\rangle = \hbar^2 \frac{1}{2}\left(\frac{1}{2}+1\right)|2, 1/2; 3/2, m_j\rangle$$
$$= \frac{3\hbar^2}{4}|2, 1/2; 3/2, m_j\rangle$$

$$H\left|2, 1/2 ; \tfrac{3}{2}, m_j\right\rangle = \left(A + \frac{B}{2}\left(\frac{15\hbar^2}{4} - 6\hbar^2 - \frac{3\hbar^2}{4}\right) + 6C\hbar^2\right)\left|2, 1/2; \tfrac{3}{2}, m_j\right\rangle$$

$$= \underbrace{\left(A + B\hbar^2 + 6C\hbar^2\right)}_{E_{j=3/2}}\left|2, 1/2, \tfrac{3}{2}, m_j\right\rangle$$

$$E_{j=3/2} = A + B\hbar^2 + 6C\hbar^2 \,//$$

- $\left|2, 1/2, \tfrac{5}{2}, m_j\right\rangle \Rightarrow$

$$H\left|2, 1/2, \tfrac{5}{2}, m_j\right\rangle = \left[A + \frac{B}{2}\left(J^2 - L^2 - S^2\right) + CL^2\right]\left|2, 1/2, \tfrac{5}{2}, m_j\right\rangle$$

$$= \left[A + \frac{B}{2}\left(\hbar^2\frac{35}{4} - 6\hbar^2 - \frac{3\hbar^2}{4}\right)\right]\left|2, 1/2, 5/2, m_j\right\rangle$$

$$= \underbrace{\left(A - \frac{3\hbar^2}{2} + 6\hbar^2\right)}_{E_{j=5/2}}\left|2, 1/2, 5/2, m_j\right\rangle$$

$$E_{j=5/2} = A - \frac{3\hbar^2}{2} + 6\hbar^2 \,//$$

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6.2. Bir manyetik alan ve birbirleriyle etkileşen iki spin sistemi için, $S_1 = 1$, $S_2 = 1/2$ için sistemin enerji özdeğerlerini ve özdurumlarını bulunuz.

$$H = \frac{A}{\hbar^2} \vec{S_1} \cdot \vec{S_2} + \frac{B}{\hbar}(S_{1z} + S_{2z})$$

$$\vec{S} = \vec{S_1} + \vec{S_2} \Rightarrow S = |\vec{S_1} - \vec{S_2}|, \dots (S_1 + S_2)$$

$$S = 3/2,\ 1/2$$

$S = 3/2 \rightarrow m_j = -3/2, -1/2, +1/2, 3/2$

$S = 1/2 \rightarrow m_j = -1/2, +1/2$

$$H = \frac{A}{\hbar^2} \underbrace{\left(\frac{S^2 - S_1^2 - S_2^2}{2} \right)}_{\vec{S_1} \cdot \vec{S_2}} + \frac{B}{\hbar} \underbrace{(S_{1z} + S_{2z})}_{S_{Tz}}$$

$S_{Tz} |S, m_j\rangle = m_j \hbar |S, m_j\rangle$

$$H = \frac{A}{\hbar^2} (S^2 - S_1^2 - S_2^2) + \frac{B}{\hbar} S_{Tz}$$

$$|S_1, S_2; S, m_j\rangle$$

- $|1, 1/2; 3/2, -3/2\rangle \Rightarrow$

$$H |1, 1/2; 3/2, -3/2\rangle = \left(\frac{A}{\hbar^2} \left[\hbar^2 \frac{3}{2}\left(\frac{5}{2}\right) - \hbar^2 1(2) - \hbar^2 \frac{1}{2}\left(\frac{3}{2}\right) \right] + \frac{B}{\hbar}\left(-\frac{3}{2}\hbar\right) \right) |1, 1/2, 3/2, -3/2\rangle$$

$$= \underbrace{\left(\frac{A}{2} - \frac{3B}{2} \right)}_{E(j = 3/2,\ m_j = -3/2)} |1, 1/2, 3/2, -3/2\rangle$$

$E(j = 3/2, m_j = -3/2) = \frac{A}{2} - \frac{3B}{2}$

$E(j = 3/2, m_j = -1/2) = \frac{A}{2} - \frac{B}{2}$

$E(j = 3/2, m_j = 1/2) = \frac{A}{2} + \frac{B}{2}$

$E(j = 3/2, m_j = 3/2) = \frac{A}{2} + \frac{3B}{2}$

$E(j = 1/2, m_j = 1/2) = -A + B/2$

$E(j = 1/2, m_j = -1/2) = -A - B/2$

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