ANALYSİS I Homework 2 06.11.2015

1. Let A be bounded above . Define $B = \{b \in \mathbb{R} : \ b \ is\ an\ upper\ bound\ for\ A\ \}$.

Show that $supA = infB$.

2. Let A be a subset of $\mathbb{R}$ and $\propto \in \mathbb{R}$.

a) $\propto = SupA \ \Leftrightarrow \ (\propto, +\infty) \cap A = \emptyset \ \ and\ for\ \forall\varepsilon > 0, \ \ (\propto - \varepsilon, \propto] \cap A \neq \emptyset$

b) $\propto = infA \ \Leftrightarrow \ (-\infty, \propto) \cap A = \emptyset \ \ and\ for\ \forall\varepsilon > 0, \ \ [\propto, \propto + \varepsilon) \cap A \neq \emptyset$

3. Let a_n $be\ a\ sequence\ of$ $\mathbb{R}$. Prove that: $a_n \to 0 \ \Leftrightarrow \ |a_n| \to 0$

4. Let a_n $be\ a\ sequence\ of$ $\mathbb{R}$ $with$ $a_n \geq 0$ $for\ all\ n \in \mathbb{N}$.

Prove that: If $a_n \to a \in \mathbb{R}$ $then$ $\sqrt{a_n} \to \sqrt{a}$

5. Let $x, y \in \mathbb{R}$ $with$ $x < y$. For $\varepsilon = \frac{y-x}{2}$,

Show that: Whether $B_\varepsilon(x) \cap B_\varepsilon(y) = \emptyset$ or $B_\varepsilon(x) \cap B_\varepsilon(y) \neq \emptyset$? Why?

6. $\forall\, n \in \mathbb{N}$ $a_n > 0$. If the sequence (a_n) is monotone, then show that the sequence

$\left(\frac{1-a_n}{a_n}\right)$ is monotone**.**

7. $\forall\, n \in \mathbb{N}$ $a_n > 0$. If the sequence (a_n) is monotone, then show that the sequence

$\left(\dfrac{1 - a_n}{1 + a_n}\right)$ is monotone.

8. Do the proof of the theorem that I didn’t do in courses….