1) Hidrojen atomunun taban durumunda en olası r değeri nedir?

Taban durumu: $n=1$, $l=0$, $m=0$ $\Rightarrow \psi_{nlm} \rightarrow \psi_{100} = R_{10} Y_0^0$

$$R_{10} = \frac{2}{a^{3/2}} e^{-r/a}, \quad Y_0^0 = \frac{1}{\sqrt{4\pi}}$$

$$\psi_{100} = \frac{2}{\sqrt{4\pi a^3}} e^{-r/a}$$

Olasılık yoğunluğu $\Rightarrow |\psi_{100}|^2 = \frac{1}{\pi a^3} e^{-2r/a}$

Olasılık; $P = \int |\psi_{100}|^2 dV = \int \frac{1}{\pi a^3} e^{-2r/a} r^2 \sin\theta \, dr \, d\theta \, d\phi$

$$P(r) = \frac{1}{\pi a^3} \int e^{-2r/a} r^2 dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi = \frac{4}{a^3} \int e^{-2r/a} r^2 dr$$

$$P(r) = \int \underbrace{\frac{4}{a^3} r^2 e^{-2r/a}}_{p(r)} dr = \int p(r) \, dr$$

$$p(r) = \frac{4}{a^3} r^2 e^{-2r/a}$$

r ile $r+dr$ arasında bulunma olasılığı!!!

• r ile $r+dr$ arasında bulunma olasılığı $p(r)$ ise, en olası r değeri için;

$$\frac{dp(r)}{dr} = 0 \Rightarrow \frac{4}{a^3}\left[2r e^{-2r/a} + r^2 \left(-\frac{2}{a}\right) e^{-2r/a}\right] = 0$$

$$\frac{8r}{a^3} e^{-2r/a}\left(1 - \frac{r}{a}\right) = 0 \Rightarrow \frac{r}{a} = 1 \Rightarrow \boxed{r = a},$$

en olası r değeri!!

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2) a) $n=3$, $l=2$ ve $m=1$ durumu için Hidrojenin uzaysal dalga fonksiyonunu kurunuz.

b) Dalga fonksiyonunun normlanmış olduğunu gösteriniz.

c) Bu durum için r^s'in beklenen değerini bulunuz. s'nin hangi değer aralığı için $\langle r^s \rangle$ sonlu kalır?

$$R_{32} = \frac{4}{81\sqrt{30}} \frac{1}{\sqrt{a^3}} \left(\frac{r}{a}\right)^2 e^{-r/3a}$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin\theta\cos\theta \, e^{+i\phi}$$

a: $\left.\begin{matrix} n=3 \\ l=2 \\ m=1 \end{matrix}\right\}$ $\psi_{nlm} = R_{nl} Y_l^m \Rightarrow \psi_{321} = R_{32} Y_2^1$

$$\Rightarrow \psi_{321} = -\frac{1}{\sqrt{\pi}} \frac{1}{81 a^{7/2}} r^2 e^{-r/3a} \sin\theta\cos\theta \, e^{+i\phi}$$

b: $\langle \psi_{321} | \psi_{321} \rangle = \int \psi_{321}^* \psi_{321} \, dv = 1$ olmalı;

$$\langle \psi_{321} | \psi_{321} \rangle = \frac{1}{\pi} \frac{1}{81^2} \frac{1}{a^7} \int_V r^4 e^{-2r/3a} \sin^2\theta\cos^2\theta \, e^{i(1-1)\phi} \, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$e^{-i\phi} e^{i\phi} = e^{i(\phi-\phi)} = e^0 = 1$$

$$\langle \psi_{321} | \psi_{321} \rangle = \frac{1}{\pi 81^2 a^7} \int_0^\infty r^6 e^{-2r/3a} dr \int_0^\pi \sin^2\theta\cos^2\theta \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$\langle \psi_{321} | \psi_{321} \rangle = \frac{2\pi}{\pi 81^2 a^7} \int_0^\infty r^6 e^{-2r/3a} dr \int_0^\pi \underbrace{\left(\frac{1-\cos 2\theta}{2}\right)}_{\sin^2\theta} \cos^2\theta \sin\theta \, d\theta$$

$$I = \int_0^\infty r^n e^{-r/b} dr = n! \, b^{n+1} \rightarrow \left.\begin{matrix} n=6 \\ b=\frac{3a}{2} \end{matrix}\right\} I = 6! \left(\frac{3a}{2}\right)^7$$

$$\langle \psi_{321} | \psi_{321} \rangle = \frac{2}{81^2 a^7} \left[6! \left(\frac{3a}{2}\right)^7\right] \left[-\frac{\cos^3\theta}{3} + \frac{\cos^5\theta}{5}\right]_0^\pi = \frac{2}{3^{\cancel{8}} \cancel{a^7}} 6.5.4.3.2 \frac{\cancel{3^7} \cancel{a^7}}{2^7} \left[\frac{2}{3} - \frac{2}{5}\right]$$

$$\langle \psi_{321} | \psi_{321} \rangle = 1 \,//$$

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c) $\langle r^s \rangle_{321} = \langle \psi_{321} | r^s | \psi_{321} \rangle = \int \psi^*_{321}\, r^s\, \psi_{321}\, dV = \int r^s |R_{32}|^2 dr \int |Y_2^1|^2 \sin\theta\, d\theta\, d\phi$

$$\Rightarrow \langle r^s \rangle = \int_0^\infty r^s |R_{32}|^2 dr \underbrace{\int |Y_2^1|^2 \sin\theta\, d\theta\, d\phi}_{1} \Rightarrow \langle r^s \rangle = \int r^s |R_{32}|^2 dr$$

$$\Rightarrow \langle r^s \rangle = \left(\frac{4}{81}\right)^2 \frac{1}{30a^7} \int_0^\infty r^{s+6} e^{-2r/3a} dr \qquad I = \int_0^\infty r^n e^{-r/b} dr = n!\,(b)^{n+1}$$

$$\left.\begin{aligned} n &= s+6 \\ b &= \tfrac{3a}{2} \end{aligned}\right\} I = (s+6)!\left(\frac{3a}{2}\right)^{s+7}$$

$$\langle r^s \rangle = \frac{8}{15(81)^2 a^7}(s+6)!\left(\frac{3a}{2}\right)^{s+7} \; //$$

$(s+6)! \rightarrow s > -7$ olmalı!!

$\boxed{-7 < s < \infty}$

3) ψ_{321} durumu için, Hidrojende r'deki belirsizliği hesaplayınız.

$$\Delta r = \sigma r = \sqrt{\langle r^2 \rangle - \langle r \rangle^2}$$

• $\langle r^s \rangle = \dfrac{8}{15(81)^2 a^7}(s+6)!\left(\dfrac{3a}{2}\right)^{s+7}$

$$s = 1 \Rightarrow \langle r \rangle = \frac{8}{15(81)^2 a^7}(1+6)!\left(\frac{3a}{2}\right)^{1+7} = \frac{8}{15(81)^2 a^7}\, 7!\, \frac{(3a)^8}{2^8}$$

$$\Rightarrow \langle r \rangle = \frac{\cancel{8}}{\cancel{3}.\cancel{5}.\cancel{3}.\cancel{3}.\cancel{3}.\cancel{3}.\cancel{3}\,\cancel{a^7}}\, 7.\overset{3}{\cancel{6}}.\cancel{5}.\cancel{4}.\cancel{3}.\cancel{2}.1\; \frac{a.\cancel{a^7}\,\cancel{3}.\cancel{3}.\cancel{3}.\cancel{3}.\cancel{3}.\cancel{3}.\cancel{3}.\cancel{3}}{\cancel{2}.\cancel{2}.\cancel{2}.2.\cancel{2}.\cancel{2}.\cancel{2}.\cancel{2}}$$

$$\boxed{\langle r \rangle = \frac{21a}{2}} \qquad \Delta r = \sqrt{126a^2 - \left(\frac{21a}{2}\right)^2} = 15{,}75a \; //$$

• $\langle r^2 \rangle \Rightarrow s = 2 \Rightarrow \langle r^2 \rangle = \dfrac{8}{15(81)^2 a^7}(2+6)!\left(\dfrac{3a}{2}\right)^{2+7}$

$$\langle r^2 \rangle = \left(\frac{21a}{2}\right) 8 \left(\frac{3a}{2}\right) = \boxed{126a^2}$$

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