ANALYSIS II Homework 3 **02.03.2016**

1. Find the derivatives of the *n*-th order of the following functions

(a) $f(x)=\ln x$ **(b)** $f(x)=\sin x$, *Answer* : $f^{(n)}(x)=\sin\left(x+\frac{n\pi}{2}\right)$

(c) $f(x)=\ln\left(x^2+x-2\right)$ **(d)** $f(x)=\frac{x}{x^2-1}$

2. Do the following functions satisfy the conditions of the Rolle theorem? If yes, find the all x_0 in the indicated interval such that $f'(x_0)=0$. If not, explain why?

(a) $f(x)=x^2$ in $[3,4]$ **(b)** $f(x)=\ln\sin x$ in $\left[\frac{\pi}{6},\frac{5\pi}{6}\right]$

(c) $f(x)=1-|x|$ in $[-1,1]$

3. Do the following functions satisfy the conditions of the Mean value theorem? If yes, find the values of x_0 appearing in this formula.

(a) $f(x)=1-\sqrt[3]{x^2}$ in $[-1,1]$ **(b)** $f(x)=\ln x$ in $[1,3]$

(c) $f(x)=4x^3-5x^2+x-2$ in $[0,1]$ **(c)** $f(x)=\sqrt[5]{x^4(x-1)}$ in $\left[-\frac{1}{2},\frac{1}{2}\right]$

4. Compute the following limits by using L'Hospital Rule.

(a) $\lim\limits_{x\to a}\arcsin\left(\frac{x-a}{a}\right)\cot(x-a)=?$ **(b)** $\lim\limits_{x\to 1}\frac{a^{\ln x}-x}{\ln x}=?\ (a>0)$

(c) $\lim\limits_{x\to 0}(\pi-2\arctan x)\ln x=?$ **(d)** $\lim\limits_{x\to a}\left(2-\frac{x}{a}\right)^{\tan\left(\pi x/2a\right)}=?(a\neq 0)$

5. Find the extrema and increasing-decreasing intervals of the following functions.

(a) $f(x)=\frac{3}{4}x^4-x^3-9x^2+7$ **(b)** $f(x)=x^4-8x^3+22x^2-24x+12$

(c) $f(x)=\sqrt[3]{x^2}-x^2$ **(d)** $f(x)=\frac{x^2-3x+2}{x^2+2x+1}$ **(e)** $f(x)=\sqrt{e^{x^2}-1}$

(f) $f(x)=x(x+1)^3(x-3)^2$ **(g)** $f(x)=\sqrt[3]{(x-1)^2}+\sqrt[3]{(x+1)^2}$

6. Find the greatest and the least values of the following functions on the indicated intervals.

(a) $f(x)=2x^3-3x^2-12x-1$ on $\left[-2,\frac{5}{2}\right]$ **(b)** $f(x)=x^2\ln x$ on $[1,e]$

(c) $f(x)=\sqrt{\left(1-x^2\right)\left(1+2x^2\right)}$ on $[-1,1]$ **(d)** $f(x)=xe^{-x}$ on $[0,+\infty)$

7. Find the intervals in which the graphs of the following functions are convex (concave up) or concave (concave down) and locate the the points of inflection.

(a) $f(x)=x^4+x^3-18x^2+24x-12$ **(b)** $f(x)=3x^4-8x^3+6x^2+12$

(c) $f(x)=4\sqrt{(x-1)^5}+20\sqrt{(x-1)^3}$ $(x\geq 1)$ **(d)** $f(x)=\frac{x}{1+x^2}$

(e) $f(x)=\frac{\ln^2 x}{x}$ $(x>0)$ **(f)** $f(x)=x\sin(\ln x)$ $(x>0)$

8. Find the asymptotes of the following curves.

(a) $f(x)=\frac{5x}{x-3}$ **(b)** $f(x)=\frac{3x}{x-1}+3x$ **(c)** $f(x)=\frac{1}{x}+4x^2$ **(d)** $f(x)=xe^{1/x}$

(e) $f(x)=\frac{3x}{2}\ln\left(e-\frac{1}{3x}\right)$ **(f)** $f(x)=\sqrt{1+x^2}+2x$ **(g)** $f(x)=2\sqrt{4+x^2}$

9. Investigate (Domain, Asymptotes, extremas, increasing-decreasin and concave-convex intervals, inflection points) and grapf the folloving functions.

(a) $f(x)=x^6-3x^4+3x^2-5$ **(b)** $f(x)=\frac{2x^3}{x^2-4}$ **(c)** $f(x)=x+\ln\left(x^2-1\right)$

(d) $f(x)=1+x^2-\frac{x^4}{2}$ **(e)** $f(x)=\frac{x^4}{(x+1)^3}$