ANALYSIS II Homework 4 **14.03.2016**

1. Determine which of the following series converges. If it converges, find its sum.

(a) $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$ **(b)** $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$ **(c)** $\sum_{n=1}^{\infty} \ln\left(1-\frac{1}{n^2}\right)$

(d) $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$ **(e)** $\sum_{n=1}^{\infty} \frac{3^{n-1}}{7^n}$ **(f)** $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$

(g) $\sum_{n=1}^{\infty} \frac{3\cdot 5^n + 5\cdot 3^n}{15^n}$ **(h)** $\sum_{n=1}^{\infty} \frac{3^n - 2^n}{6^n}$ **(i)** $\sum_{n=1}^{\infty} \left(\sqrt{5}\right)^{1-n}$

2. Determine which of the following series converges.

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2+n+2}$ **(b)** $\sum_{n=1}^{\infty} \frac{n^3+1}{n^4+1}$ **(c)** $\sum_{n=1}^{\infty} \frac{n+1}{n+5}$ **(d)** $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

(e) $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$ **(f)** $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$ **(g)** $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$ **(h)** $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$

(i) $\sum_{n=1}^{\infty} \frac{n+1}{n\cdot 6^n}$ **(j)** $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ **(k)** $\sum_{n=1}^{\infty} \frac{5^n}{n(n!)}$ **(l)** $\sum_{n=1}^{\infty} 5^n \sin(\frac{1}{6^n})$

(i) $\sum_{n=1}^{\infty} \frac{1}{n(1+\sin n)}$ **(j)** $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt[3]{n^7-n^4+1}}$ **(k)** $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt[3]{n^6+1}}$ **(l)** $\sum_{n=1}^{\infty} \sqrt{n}\sin(\frac{1}{n})$

3. Do the following functions satisfy the conditions of the Mean value theorem? If yes, find the values of x_0 appearing in this formula.

(a) $f(x) = 1-\sqrt[3]{x^2}$ in $[-1,1]$ **(b)** $f(x) = \ln x$ in $[1,3]$

(c) $f(x) = 4x^3 - 5x^2 + x - 2$ in $[0,1]$ **(c)** $f(x) = \sqrt[5]{x^4(x-1)}$ in $\left[-\frac{1}{2}, \frac{1}{2}\right]$

4. Compute the following limits by using L'Hospital Rule.

(a) $\lim_{x\to a} \arcsin\left(\frac{x-a}{a}\right)\cot(x-a) = ?$ **(b)** $\lim_{x\to 1} \frac{a^{\ln x} - x}{\ln x} = ?$ $(a>0)$

(c) $\lim\limits_{x\to 0}(\pi-2\arctan x)\ln x=?$ **(d)** $\lim\limits_{x\to a}\left(2-\frac{x}{a}\right)^{\tan\left(\pi x/2a\right)}=?\,(a\neq 0)$

5. Find the extrema and increasing-decreasing intervals of the following functions.

(a) $f(x)=\frac{3}{4}x^4-x^3-9x^2+7$ **(b)** $f(x)=x^4-8x^3+22x^2-24x+12$

(c) $f(x)=\sqrt[3]{x^2}-x^2$ **(d)** $f(x)=\frac{x^2-3x+2}{x^2+2x+1}$ **(e)** $f(x)=\sqrt{e^{x^2}-1}$

(f) $f(x)=x(x+1)^3(x-3)^2$ **(g)** $f(x)=\sqrt[3]{(x-1)^2}+\sqrt[3]{(x+1)^2}$

6. Find the greatest and the least values of the following functions on the indicated intervals.

(a) $f(x)=2x^3-3x^2-12x-1$ on $\left[-2,\frac{5}{2}\right]$ **(b)** $f(x)=x^2\ln x$ on $[1,e]$

(c) $f(x)=\sqrt{(1-x^2)(1+2x^2)}$ on $[-1,1]$ **(d)** $f(x)=xe^{-x}$ on $[0,+\infty)$

7. Find the intervals in which the graphs of the following functions are convex (concave up) or concave (concave down) and locate the the points of inflection.

(a) $f(x)=x^4+x^3-18x^2+24x-12$ **(b)** $f(x)=3x^4-8x^3+6x^2+12$

(c) $f(x)=4\sqrt{(x-1)^5}+20\sqrt{(x-1)^3}\quad(x\geq 1)$ **(d)** $f(x)=\frac{x}{1+x^2}$

(e) $f(x)=\frac{\ln^2 x}{x}\quad(x>0)$ **(f)** $f(x)=x\sin(\ln x)\quad(x>0)$

8. Find the asymptotes of the following curves.

(a) $f(x)=\frac{5x}{x-3}$ **(b)** $f(x)=\frac{3x}{x-1}+3x$ **(c)** $f(x)=\frac{1}{x}+4x^2$ **(d)** $f(x)=xe^{1/x}$

(e) $f(x)=\frac{3x}{2}\ln\left(e-\frac{1}{3x}\right)$ **(f)** $f(x)=\sqrt{1+x^2}+2x$ **(g)** $f(x)=2\sqrt{4+x^2}$

9. Investigate (Domain, Asymptotes, extremas, increasing-decreasin and concave-convex intervals, inflection points) and grapf the folloving functions.

(a) $f(x)=x^6-3x^4+3x^2-5$ **(b)** $f(x)=\frac{2x^3}{x^2-4}$ **(c)** $f(x)=x+\ln(x^2-1)$

(d) $f(x)=1+x^2-\frac{x^4}{2}$ **(e)** $f(x)=\frac{x^4}{(x+1)^3}$