Differential Equations II Homework 1 **14.03.2017**

1. Find the general solution of the given systems

$$\frac{dy}{dx} = Ay, \quad where \quad A = \left(a_{ij}\right)_{nxn}, \quad \frac{dy}{dx} = \begin{bmatrix} \frac{dy_1}{dx} \\ \frac{dy_2}{dx} \\ \vdots \\ \frac{dy_n}{dx} \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

(a) $A = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}$ **(b)** $A = \begin{bmatrix} -2 & 3 \\ -1 & 2 \end{bmatrix}$ **(c)** $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ **(d)** $A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}$

(e) $A = \begin{bmatrix} 1 & -5 & 0 \\ 1 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **(f)** $A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 1 & 2 \\ 1 & -1 & 4 \end{bmatrix}$ **(g)** $A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 1 & 2 \\ 1 & -1 & 4 \end{bmatrix}$ **(h)** $A = \begin{bmatrix} 1 & 3 & 2 \\ -1 & 2 & 1 \\ 4 & -1 & -1 \end{bmatrix}$

(i) $A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 1 & 2 \\ 1 & -1 & 4 \end{bmatrix}$ **(j)** $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & -2 \\ 2 & 2 & 1 \end{bmatrix}$ **(k)** $A = \begin{bmatrix} -7 & 0 & 6 \\ 0 & 5 & 0 \\ 6 & 0 & 2 \end{bmatrix}$ **l)** $A = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & -1 & 2 \end{bmatrix}$

2. Solve the given initial valure problems.

(a) $A = \begin{bmatrix} 4 & 5 \\ -2 & -2 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ **(b)** $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ **(d)** $A = \begin{bmatrix} 1 & -1 \\ 5 & -3 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(e) $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ **(f)** $A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 1 & 2 \\ 1 & -1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

(g) $A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 2 & 3 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ **(h)** $A = \begin{bmatrix} -1 & 1 & 2 \\ -1 & 1 & 1 \\ -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad y(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(i) $A=\begin{bmatrix}-3 & 0 & 2\\ 1 & -1 & 0\\ -2 & -1 & 0\end{bmatrix}$, $y(0)=\begin{bmatrix}0\\ -1\\ -2\end{bmatrix}$ **(j)** $A=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & -1\\ 3 & 3 & -1\end{bmatrix}$, $y(0)=\begin{bmatrix}1\\ -2\\ -1\end{bmatrix}$

k) $A=\begin{bmatrix}1 & -3 & 2\\ 0 & -1 & 0\\ 0 & -1 & -2\end{bmatrix}$, $y(0)=\begin{bmatrix}-2\\ 0\\ 3\end{bmatrix}$ **l)** $A=\begin{bmatrix}0 & 2 & 0 & 0\\ -2 & 0 & 0 & 0\\ 0 & 0 & 0 & -3\\ 0 & 0 & 3 & 0\end{bmatrix}$, $y(0)=\begin{bmatrix}1\\ 1\\ 1\\ 0\end{bmatrix}$

3. Find power series solutions in power of x of the differential equations.

(a) $y''+xy'+y=0$ **(b)** $y''+8xy'-4y=0$ **(c)** $y''+xy'+\left(2x^2+1\right)y=0$

(d) $y''+xy'+(x-4)y=0$ **(e)** $y''+xy'+(3x+2)y=0$ **(f)** $y''-xy'+(3x-2)y=0$

g) $\left(x^2+1\right)y''+xy'+xy=0$ **h)** $(x-1)y''-(3x-2)y'+2xy=0$

i) $\left(x^2+1\right)y''+xy'+xy=0$

4. Find power series solutions in power of x of the initial-value problems.

(a) $y''-xy'-y=0,\ y(0)=1,\ y'(0)=0$ **(b)** $y''+xy'-2y=0,\ y(0)=0,\ y'(0)=1$

(c) $\left(x^2+1\right)y''+xy'+2xy=0,\ y(0)=2,\ y'(0)=3$

(d) $\left(2x^2-3\right)y''-2xy'+y=0,\ y(0)=-1,\ y'(0)=5$

5. Locate and classify the singular points of each of the differential equations.

(a) $\left(x^2-3x\right)y''+(x+2)y'+y=0$ **(b)** $\left(x^3+x^2\right)y''+\left(x^2-2x\right)y'+4y=0$

(c) $\left(x^4+2x^3+x^2\right)y''+2(x-1)y'+x^2y=0$

(d) $\left(x^5+x^4-6x^3\right)y''+x^2y'+(x-2)y=0$

6. Locate and classify the singular points of each of differential equations and find power series solutions near $x = 0$.

(a) $2x^2 y'' + xy' + \left(x^2 - 1\right) y = 0$

(b) $2x^2 y'' + xy' + \left(2x^2 - 3\right) y = 0$

(c) $x^2 y'' - xy' + \left(x^2 + \frac{8}{9}\right) y = 0$

(d) $2xy'' + y' + 2y = 0$

(e) $3xy'' - (x - 2) y' - 2y = 0$

(f) $xy'' + 2y' + xy = 0$

(g) $xy'' - \left(x^2 + 2\right) y' + xy = 0$

(f) $x^2 y'' + xy' + (x - 1) y = 0$

Y. Doç. Dr. Cemal ÇİÇEK