ANALYSIS II Homework 1 17.02.2016

1. $f:[1,2] \to \mathbb{R}$ is given by $f(x) = -x^3 + 3cos\left(\frac{\pi x}{2}\right)$. Is there at least one x_0 in $(1,2)$ such that $f(x_0) = 0$? Explain your answer. **(II. Öğretim sorumlu)**

2. (a) Find the tangent line and normal line of the curve $x^3 + y^3 - xy - 7 = 0$ at the point $(1,2)$

(b) Find the tangent line and normal line of the curve $y = x^3 + 2x^2 - 4x - 3$ at the point $(-2,5)$

(c) Find the tangent line and normal line of the curve $y = \sqrt[3]{x-1}$ at the point $(1,0)$

(d) Write the equations of the tangent lines and the normal lines to the curve $y = (x-1)(x-2)(x-3)$ at the points of its intersection with the $x-$axis.

3. Find the limits by using L'Hospital rule:

(a) $\underset{x\to 0}{Lim}\left(1+x^2\right)^{1/(x^2)} = ?$ **(b)** $\underset{x\to 0}{Lim}\left(\frac{e^x}{x} - \frac{1}{\sin x}\right) = ?$

(c) $\lim\limits_{x\to\pi}(x-\pi)\tan\left(\frac{x}{2}\right)$ **(d)** $\lim\limits_{x\to 0}\frac{\tan x}{\ln(x-1)}$

(e) $\lim\limits_{x\to\pi}\frac{1+Cos(x)}{(x-\pi)^2}$ **(f)** $\lim\limits_{x\to 1}\frac{1-\sin(\frac{\pi}{2}x)}{1-x} = ?$

(g) $\lim\limits_{x\to \pi/4}\frac{\sin x - \cos x}{\tan x - 1}$ **(h)** $\lim\limits_{x\to 0}\frac{1-\sqrt{\cos x}}{1-\cos\sqrt{x}}$

4. If $f(x)$ is defined on interval (a,b) and attains a minimum value at $x_0 \in (a,b)$, and if $f'(x_0)$ exists, then $f'(x_0) = 0$.

5. Do the following functions satisfy the conditions of the Rolle Theorem on the indicated closed intervals? If yes, find the all points x_0 which figure in the Rolle Formula.

(a) $f(x) = (2x-3)^2(4x+1),\quad [0,2]$ **(b)** $f(x) = \sqrt[3]{(x-2)^2},\quad [0,4]$

(c) $f(x) = (2x-3)^2(4x+1),\quad [0,2]$ **(d)** $f(x) = \tan x,\quad [0,\pi]$

(e) $f(x) = (2x-3)^2(4x+1),\quad [0,2]$ **(f)** $f(x) = \tan x,\quad [0,\pi]$

6. Find the dervatives of the following functions by using derivatives table.

(a) $f(x)=\dfrac{\arctan\left(xe^{x}\right)}{1+e^{x}}$ **(b)** $f(x)=\dfrac{\arcsin x}{\arccos x}$

(c) $f(x)=\sqrt[3]{x\ln x+\cos x}$ **(d)** $f(x)=5^{x^{2}\tan x}\sin\left(x^{2}+\tan x\right)$

(e) $f(x)=\dfrac{15}{4(x-3)^{4}}+\dfrac{10}{3(x-3)^{3}}+\dfrac{1}{2(x-3)^{2}}$

7. (a) The function $f(x)=x(x+1)(x+2)(x+3)$ is given.

Show that the equation $f^{/}(x)=0$ has three real roots.

(b) $x=0$ obviously is a root of the equation $e^{x}-x-1=0$.

Show that this equation can not have any other real root.

(c) Show that the equation $x^{5}+2x^{3}+5x-10=0$ has one and only one real root on $[0,1]$.

(d) Show that the equation $x\arctan x-1=0$ has one and only one real root on $\left[1,\dfrac{3}{2}\right]$.

8. Prove the following inequalities.

(a) $\dfrac{x}{1+x^{2}}<\arctan x<x$ for $x>0$. $\left(\text{Hint. Take } f(x)=\arctan x \text{ on } [0,x]\right)$

(b) $1<\dfrac{x}{\sin x}<\dfrac{\pi}{2}$ for $0<x<\dfrac{\pi}{2}$. $\left(\text{Hint. Take } f(x)=\sin x \text{ on } [0,x]\right)$

(c) $\dfrac{3}{25}+\dfrac{\pi}{4}<\arctan\dfrac{4}{3}<\dfrac{\pi}{4}+\dfrac{1}{6}$. $\left(\text{Hint. Take } f(x)=\arctan x \text{ on } \left[1,\dfrac{4}{3}\right]\right)$

8. Find the dervatives $y^{/}$ of te following $y=y(x)$ functions at the indicated points.

(a) $(x+y)^{3}=27(x-y)$ at the point $(2,1)$

(b) $ye^{y}=e^{x+1}$ at the point $(0,1)$

(c) $y^{2}=x+\ln\left(\dfrac{y}{x}\right)$ at the point $(1,1)$

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