

1) Bir parçacık $t=0$ anında,

$$\psi(x, t=0) = \begin{cases} A(a^2 - x^2), & -a \leq x \leq a \\ 0, & \text{diğer yerlerde} \end{cases}$$

ile verilen dalgın fonksiyonu ile temsil edilmektedir.

- $\psi(x, 0)$ 'i normalize edin.
- $t=0$ anında x 'in beklenen değeri bulun.
- $t=0$ anında P 'nin beklenen değeri bulun.
- x^2 'nin beklenen değeri bulun.
- P^2 'nin beklenen değeri bulun.

$$\begin{aligned} \text{a: } 1 &= \langle \psi(x, 0) | \psi(x, 0) \rangle = \int_{-a}^{+a} |\tilde{\psi}|^2 dx = \int_{-a}^{-a} |\tilde{\psi}|^2 dx + \int_{-a}^{+a} |\tilde{\psi}|^2 dx + \int_{+a}^{+a} |\tilde{\psi}|^2 dx \\ 1 &= \int_{-a}^{+a} A^* (a^2 - x^2)^* A (a^2 - x^2) dx = |A|^2 \int_{-a}^{+a} (a^2 - x^2) dx = |A|^2 \int_{-a}^{+a} (a^2 - 2a^2 x^2 + x^4) dx \\ \Rightarrow \frac{1}{|A|^2} &= \left[a^4 x - \frac{2a^2 x^3}{3} + \frac{x^5}{5} \right]_{-a}^{+a} \Rightarrow \frac{1}{|A|^2} = \left[a^5 - \frac{2a^5}{3} + \frac{a^5}{5} - a^5 + \frac{2a^5}{3} - \frac{a^5}{5} \right] \end{aligned}$$

$$A = \sqrt{\frac{15}{16a^5}} //$$

$$\text{b: } \langle x \rangle = \langle \psi | x | \psi \rangle = |A|^2 \int_{-a}^{+a} \underbrace{(a^2 - x^2)}_{\text{teğel fonksiyonu}} x dx = |A|^2 \int_{-a}^{+a} (a^2 x - 2a^2 x^3 + x^5) dx$$

$$\langle x \rangle = \frac{15}{16a^5} \left[a^4 \frac{x^2}{2} - 2a^2 \frac{x^4}{4} + \frac{x^6}{6} \right]_{-a}^{+a} \Rightarrow \langle x \rangle = 0 //$$

$$\text{c: } \langle P \rangle = \langle \psi | P | \psi \rangle = \langle \psi | \underbrace{(-i\hbar \frac{d}{dx})}_{P} | \psi \rangle = |A|^2 \int_{-a}^{+a} (a^2 - x^2) \left(-i\hbar \frac{d}{dx} \right) (a^2 - x^2) dx$$

$$\langle P \rangle = \frac{15}{16a^5} \int_{-a}^{+a} (a^2 - x^2) \left[-i\hbar (-2x) \right] dx = \frac{15}{8a^5} i\hbar \int_{-a}^{+a} (a^2 x - x^3) dx$$

$$\langle P \rangle = \frac{15 i\hbar}{8a^5} \left[a^2 \frac{x^2}{2} - \frac{x^4}{4} \right]_{-a}^{+a} = \frac{15 i\hbar}{8a^5} \left[\frac{a^4}{2} - \frac{a^4}{4} - \left(\frac{a^4}{2} - \frac{a^4}{4} \right) \right] \Rightarrow \langle P \rangle = 0 //$$

d:

$$\langle x^2 \rangle = \langle \psi | x^2 | \psi \rangle = |A|^2 \int_{-a}^{+a} (a^2 - x^2)^2 x^2 dx = \frac{15}{16a^5} \int_{-a}^{+a} (a^4 - 2a^2 x^2 + x^4) x^2 dx$$

$$\langle x^2 \rangle = \frac{15}{16a^5} \int_{-a}^{+a} \left[a^4 x^2 - 2a^2 x^4 + x^6 \right] dx = \frac{15}{16a^5} \left[a^4 \frac{x^3}{3} - 2a^2 \frac{x^5}{5} + \frac{x^7}{7} \right]_{-a}^{+a}$$

$$\langle x^2 \rangle = \frac{15}{16a^5} \left[\left(\frac{a^7}{3} - \frac{2a^7}{5} + \frac{a^7}{7} \right) - \left(-\frac{a^7}{3} + \frac{2a^7}{5} - \frac{a^7}{7} \right) \right] = \frac{15}{16a^5} 2 \left(\frac{a^7}{3} + \frac{2a^7}{5} - \frac{a^7}{7} \right)$$

$$\langle x^2 \rangle = \frac{a^2}{7} //$$

e:

$$\langle P^2 \rangle = \langle \psi | P^2 | \psi \rangle = \langle \psi | \left(-\hbar^2 \frac{d^2}{dx^2} \right) | \psi \rangle = |A|^2 \int_{-a}^{+a} (a^2 - x^2) \left(-\hbar^2 \frac{d^2}{dx^2} \right) (a^2 - x^2) dx$$

$$\langle P^2 \rangle = \frac{-15\hbar^2}{16a^5} \int_{-a}^{+a} (a^2 - x^2) \frac{d}{dx} (-2x) dx = \frac{15\hbar^2}{8a^5} \int_{-a}^{+a} (a^2 - x^2) (1) dx$$

$$\langle P^2 \rangle = \frac{15\hbar^2}{8a^5} \left[a^2 x - \frac{x^3}{3} \right]_{-a}^{+a} = \frac{15\hbar^2}{8a^5} \left[a^3 - \frac{a^3}{3} - \left(-a^3 + \frac{a^3}{3} \right) \right] = \frac{15\hbar^2}{4a^5} \left(a^3 - \frac{a^3}{3} \right)$$

$$\langle P^2 \rangle = \frac{15\hbar^2}{2a^5} \frac{a^3}{3} \Rightarrow \langle P^2 \rangle = \frac{5\hbar^2}{2a^2} //$$

2) Küresel simetriye sahip bir $V(r)$ potansiyeli etkisi altında hareket eden bir tane cisim dalgı fonksiyonu, A ve α birer sabit olmak üzere, $\psi(r) = A e^{-\alpha r}$ ile verilmektedir. $V(r)$ potansiyelini ve tane cisim toplam enerjisini bir sabit fonksiyon ile belirleyiniz.

$$H|\psi(r)\rangle = E|\psi(r)\rangle \Rightarrow \left[\frac{p^2}{2m} + V(r) \right] |\psi(r)\rangle = E|\psi(r)\rangle$$

$$p = -i\hbar \nabla \Rightarrow p^2 = -\hbar^2 \nabla^2 = -\hbar^2 \left[\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{d}{d\theta} \left(\sin^2 \theta \frac{d}{d\theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{d^2}{d\phi^2} \right]$$

$$\psi(r) \Rightarrow \nabla^2 = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr} \right) + 0 + 0$$

$$\Rightarrow -\frac{\hbar^2}{2m} \left[\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\psi(r)}{dr} \right) \right] + V(r)\psi(r) = E\psi(r)$$

$$\Rightarrow -\frac{\hbar^2}{2m} \left[\frac{1}{r^2} \frac{d}{dr} \left(r^2 A (-\alpha r e^{-\alpha r}) \right) \right] + (V(r) - E)\psi(r) = 0$$

$$\Rightarrow \frac{\hbar^2}{2m} \left[\frac{1}{r^2} \frac{d}{dr} \left(-A \alpha r^3 e^{-\alpha r} \right) \right] + (E - V(r))\psi(r) = 0$$

$$\Rightarrow \frac{\hbar^2}{2m} \left[\frac{1}{r^2} \left(-A \alpha 6r^2 e^{-\alpha r} - A \alpha r^3 (-\alpha r) e^{-\alpha r} \right) \right] + (E - V(r))\psi = 0$$

$$\Rightarrow -\frac{\hbar^2 A \alpha}{m r} \left[3r e^{-\alpha r} - 2r^3 \alpha e^{-\alpha r} \right] + (E - V(r))\psi = 0$$

$$\Rightarrow -\frac{\hbar^2 \alpha}{m} \left[3 A e^{-\alpha r} - 2r^2 \alpha A e^{-\alpha r} \right] + (E - V(r))\psi = 0$$

$$\Rightarrow \frac{\hbar^2}{2m} \left[-2\alpha (3 - 2\alpha r^2) \right] + E - V = 0$$

$$\Rightarrow V(r) = E - \frac{\hbar^2 \alpha}{m} (3 - 2\alpha r)$$

$$V(r) = \frac{\hbar^2 \alpha^2}{m} r^2 \left(E - \frac{3\hbar^2 \alpha}{m} \right) \quad E = C + \frac{3\hbar^2 \alpha}{m}$$

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3) $\psi(x, y, z) = A e^{-1/2(x^2 + y^2 + z^2)}$ dalgası fonksiyonu ile temsil edilen taneceğin R yarıçaplı bir kürede bulunma olasılığını hesaplayınız.

$\psi(x, y, z)$ 'si normalizeleyelim;

$$1 = \langle \psi | \psi \rangle = \int_{x=0}^{+\infty} \int_{y=0}^{+\infty} \int_{z=0}^{+\infty} |A|^2 e^{-(x^2 + y^2 + z^2)} \frac{dx dy dz}{dV}$$

$$\frac{1}{|A|^2} = \int_{-\infty}^{+\infty} e^{-x^2} dx \int_{-\infty}^{+\infty} e^{-y^2} dy \int_{-\infty}^{+\infty} e^{-z^2} dz \quad \int_0^{\infty} x^n e^{-x/\alpha} dx = \sqrt{\pi} \frac{(n)!}{n!} \left(\frac{\alpha}{\sqrt{\pi}}\right)^{n+1}$$

$$I = \int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi} \quad \int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2} \quad \int_0^{\infty} x^n e^{-x/\alpha} dx = \sqrt{\pi} \frac{(n)!}{n!} \left(\frac{\alpha}{\sqrt{\pi}}\right)^{n+1}$$

$n=0$
 $\alpha=1$

$$\Rightarrow \frac{1}{|A|^2} = \sqrt{\pi} \sqrt{\pi} \sqrt{\pi} \Rightarrow A = \pi^{-3/4} //$$

veya, küresel koordinatlara dönüştürerek;

$$\psi(x, y, z) \rightarrow \psi(r, \theta, \phi) = A e^{-r^2/2}, \quad x^2 + y^2 + z^2 = r^2$$

$$\Rightarrow 1 = \langle \psi | \psi \rangle = |A|^2 \int_0^{\infty} e^{-r^2} r^2 \sin\theta dr d\theta d\phi = |A|^2 \int_0^{\infty} r^2 e^{-r^2} dr \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi$$

4π

$$\Rightarrow \frac{1}{|A|^2} = 4\pi \int_0^R r^2 e^{-r^2} dr = 4\pi \sqrt{\pi} \left(\frac{1}{\sqrt{\pi}}\right)^{2+1+1} \frac{(2+1)!}{1!} = 4\pi \sqrt{\pi} \frac{1}{8} \cdot 2$$

$n=1$
 $l=1$

$$A = \pi^{-3/4} //$$

$$P(0 < r < R) = |A|^2 \int_0^R \int_0^\pi \int_0^{2\pi} e^{-r^2} r^2 \sin\theta dr d\theta d\phi = \pi^{-3/2} \int_0^R r^2 e^{-r^2} dr \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi$$

4π

$$P(0 < r < R) = 4\pi^{-1/2} \int_0^R r^2 e^{-r^2} dr$$

$$u = r \quad du = dr$$

$$dv = r e^{-r^2} dr$$

$$v = -\frac{e^{-r^2}}{2}$$

$$\int u dv = uv - \int v du$$

$$P(0 < r < R) = \frac{4}{\sqrt{\pi}} \left[r \left(-\frac{e^{-r^2}}{2}\right) \Big|_0^R + \int_0^R \frac{e^{-r^2}}{2} dr \right] = \frac{4}{\sqrt{\pi}} \left[-\frac{R e^{-R^2}}{2} + 0 + \int_0^R \frac{e^{-r^2}}{2} dr \right]$$

$$P(0 < r < R) = \frac{4}{\sqrt{\pi}} \left[\frac{-R e^{-R^2}}{2} + \int_0^R \frac{e^{-r^2}}{2} dr \right]$$

$\int_0^R e^{-r^2} dr$ } serise
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