Differential Equations I Homework 4 16.12.2016

1. Find the general solution of the **Cauchy-Euler** differential equations.

(a) $x^3 \dfrac{d^3y}{dx^3} - 3x^2 \dfrac{d^2y}{dx^2} + 7x\dfrac{dy}{dx} - 8y = x\ln x + \dfrac{1}{x}$

(b) $x^3 \dfrac{d^3y}{dx^3} + 3x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + 8y = \cos(\ln x)$

(c) $x^3 \dfrac{d^3y}{dx^3} + 2x^2 \dfrac{d^2y}{dx^2} = x + \sin(\ln x)$

(d) $x^3 \dfrac{d^3y}{dx^3} - x^2 \dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} - 2y = x^2 + \dfrac{1}{x}$

(e) $x^2 \dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + 2y = 20\sin(\ln x)$

(f) $x^2 \dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 3y = 2x^4e^x$

(g) $x^2 \dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} + 3y = (x-1)\ln x$

(h) $x^3 \dfrac{d^3y}{dx^3} - 2x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + 4y = \sin(\ln x)$

(i) $x^3 \dfrac{d^3y}{dx^3} + 2x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = \dfrac{1}{x}$

2. Find the general solution of the differential equation by using **variation of parameters.**

(a) $\dfrac{d^2y}{dx^2}-2y=e^{-x}\sin 2x$

(b) $\dfrac{d^2y}{dx^2}+9y=\text{cosec}2x$

(c) $9\dfrac{d^2y}{dx^2}+y=\tan^2\dfrac{x}{3}$

(d) $4\dfrac{d^2y}{dx^2}-4\dfrac{dy}{dx}+y=e^{\frac{x}{2}}\ln x$

(e) $\dfrac{d^2y}{dx^2}+y=\text{cosec}^3 x$

(f) $\dfrac{d^2y}{dx^2}+4y=\sec^2 2x$

(g) $\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+5y=e^{-2x}\sec x$

(h) $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+5y=e^{x}\tan 2\text{x}$

(i) $\dfrac{d^2y}{dx^2}+6\dfrac{dy}{dx}+9y=\dfrac{e^{-3x}}{x^3}$

(j) $\dfrac{d^2y}{dx^2}+y=\dfrac{1}{1+\sin x}$

3. Solve the initial value problems:

(a) $y''-4y'+3y=9x^2+4 \quad y(0)=6\,,\;\; y'(0)=8$

(b) $y''+4y'+13y=18e^{-2x}+4 \quad y(0)=0\,,\;\; y'(0)=4$

(c) $y''-2y'+y=2xe^{2x}+6e^{x} \quad y(0)=1\,,\;\; y'(0)=0$

(d) $y''-2y'+y=2xe^{2x}+6e^{x} \quad y(0)=1\,,\;\; y'(0)=0$

(e) $x^2y''-2xy'-10y=0 \quad y(1)=5\,,\;\; y'(1)=4$

(f) $x^2y''-2y=4x-8 \quad y(1)=4\,,\;\; y'(1)=-1$

(g) $x^2y''-4xy'+4y=4x^2-6x^3 \quad y(2)=4\,,\;\; y'(2)=-1$

(h) $x^2y''-6y=\ln x \quad y(1)=\dfrac{1}{6}\,,\;\; y'(1)=-\dfrac{1}{6}$

4*. (a) $\left(x^2+1\right)\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+2y=0$ is given.

(i) Find a particular solution in the form $y=x^n$ of the given differential equation.

(ii) By using **(i)** find general solution of the given differential equation.

(b) $(x+1)^2\dfrac{d^2y}{dx^2}-3(x+1)\dfrac{dy}{dx}+3y=0$ is given.

(i) Find a particular solution in the form $y=ax+b$ of the given differential equation.

(ii) By using **(i)** find general solution of the given differential equation.

(c) $\left(x^2-1\right)\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+2y=0$ is given.

(i) Find a particular solution in the form $y=x^n$ of the given differential equation.

(ii) By using **(i)** find general solution of the given differential equation.

(d) $(2x+1)\dfrac{d^2y}{dx^2}-4(x+1)\dfrac{dy}{dx}+4y=0$ is given.

(i) Find a particular solution in the form $y=e^{mx}$ of the given differential equation.

(ii) By using **(i)** find general solution of the given differential equation.

(e) $\left(x^3-x^2\right)\dfrac{d^2y}{dx^2}-\left(x^3+2x^2-2x\right)\dfrac{dy}{dx}+\left(2x^2+2x-2\right)y=0$ is given.

(i) Find a particular solution in the form $y=x^n$ of the given differential equation.

(ii) By using **(i)** find general solution of the given differential equation