Differential Equations I Homework II 10.10.2016

1. Find the general solution of the homogeneous differential equation

$$\frac{dy}{dx} = \frac{y}{2x} + \frac{y^2}{x^2}$$ by using the substitution $y = ux$. ***(midterm)***

2. Find the general solution of the differential equation ***(midterm)***

$$\frac{dy}{dx} = \frac{x-y+1}{y-x+1}$$

3. (a) Is the differential equation

$$(2e^y - 3xsiny)dx + (xe^y - x^2cosy)dy = 0$$ exact?

(b) If not exact, find an integrating factor $\mu = \mu(x)$.

(c) Find the solution of the given differential equation ***.*** ***(midterm)***

4. Find the general solution of the Bernoulli equation

$$\frac{dy}{dx} - \frac{y}{x} = y^2 xsinx$$ ***(midterm) Not responsible for midterm***

5. Find the general and singular(if exist) solutions of the Clairaut equation

$y = xp - \sqrt{p}$, where $p = \frac{dy}{dx}$ ***(midterm) Not responsible for midterm***

6. (a) Verify that the primitive $y = c_1 + c_2 e^{-x}$ contains two independent parameters.

(b) Find the second order differential equation of which this function is the general slution. ***(midterm)***

1. Test the following differential equations for exactness and solve those if they are exact.

(a) $\left(3x^2y+xy^2+e^x\right)dx+\left(x^3+x^2y+\sin y+e^x\right)dy=0$

(b) $\left(y\cos x-2\sin y+xy^2+e^x\right)dx-\left(2x\cos y-\sin x\right)dy=0$

(c) $\dfrac{2xy-1}{y}dx+\dfrac{x+3y}{y^2}dy=0$

(d) $\left(\dfrac{x}{y}\cos\dfrac{x}{y}+\sin\dfrac{x}{y}+\cos x\right)dx-\dfrac{x^2}{y^2}\cos\dfrac{x}{y}dy=0$

(e) $\left(\dfrac{2x^2}{x^2+y^2}+\ln\left(x^2+y^2\right)\right)dx+\dfrac{2xy}{x^2+y^2}dy=0$

(f) $\left(ye^{xy}+2xy\right)dx+\left(xe^{xy}+x^2\right)dy=0$

2. Show that every separable equation of the form

$$F(x)+G(y)\frac{dy}{dx}=0 \qquad \text{is exact.}$$

3. Find the all functions $f(x)$ such that the differential equation

$$y^2\sin x+yf(x)\frac{dy}{dx}=0 \qquad \text{is exact.}$$

Solve the differential equation for these $f(x)$.

4. The differential equation Find the all functions $f(x)\dfrac{dy}{dx}+x^2+y=0$ is known to have an integrating factor $\mu=\mu(x)$. Find the all possible functişons $f(x)$.

5. Test the following differential equations for exactness. If not exact, find an integrating factor as indicated. Then find the solution .

(a) $\left(3x^2-y^2+3\right)dx+2xydy=0,\ \mu=\mu(x)$ **(b)** $(x-2y)dx+ydy=0,\ \mu=\mu(x-y)$

(c) $\left(xy^2-y^3\right)dx+\left(1-xy^2\right)dy=0,\ \mu=\mu(y)$ **(e)** $\left(y-xy^3\right)dx+xdy=0,\ \mu=\mu(xy)$

(d) $y\left(1+2xy-x^2y^2\right)dx+x(1+2xy)dy=0,\ \mu=\mu(xy)$

(f) $\left(2xy-y^2-y\right)dx+\left(2xy-x^2-x\right)dy=0,\ \mu=\mu(x+y)$

6. Find the general solution of the linear differential equations.

(a) $x\dfrac{dy}{dx}+2y=x^2$ **(b)** $\dfrac{dy}{dx}-xy=e^{x^2/2}\cos x$ **(c)** $\dfrac{dy}{dx}+2xy=2xe^{-x^2}$

(d) $\dfrac{dx}{dy}+x=e^{-y}$ **(e)** $x\dfrac{dy}{dx}+(1+x)y=e^x$ **(f)** $x^2\dfrac{dy}{dx}+\left(x^2+2x\right)y=1$

(g) $(x+1)\dfrac{dy}{dx}+2y=e^x(x+1)^{-1}$ **(h)** $x\dfrac{dy}{dx}=2xe^{3x}+y(3x+2)$

7. Find the initial value solution of the linear differential equations.

(a) $\left(x^2+1\right)\dfrac{dy}{dx}+2xy=x^2,\ \ y(0)=-1$ **(b)** $\left(x^2-1\right)\dfrac{dy}{dx}+4y+\left(x^2-1\right)^2=0,\ \ y(0)=-6$

Yrd.Doç.Dr. Cemal ÇİÇEK