**ANALYSIS II** **Homework 7** **04.05.2016**

**1.** Find the area bounded by the curves:

**(a)** $y=\frac{x^2}{4}$, $y=\frac{x}{2}+2$ **(b)** $y^2=2x$, $x-y=4$ **(c)** $y=x^3-12x$, $y=x^2$

**(d)** $y=\sqrt{x}$, $y=\sqrt{2x}$, $y=x$ **(e)** $y=x^2-2$, $y=|x|$ **(f)** $y=\cos x$, $y=x+1$, $y=0$

**(g)** $y=x^4-2x^2$, $y=2x^2$ **(h)** $y=x^3$, $y=x^2$ **(i)** $x=y^3-4y$, $x=4-y^2$

**(j)** $y=x^3$, $y=2-x^2$, $y=0$ **(k)** $y=\frac{x^2}{3}$, $y=4-\frac{2x^2}{3}$ **(l)** $y=\frac{1}{x^2+1}$, $2y=x^2$

**(m)** $y=\sin x$, $\cos x$, $x=\frac{\pi}{4}$, $x=\frac{5\pi}{4}$ **(k*)** $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $x=2a$

**(l*)** $x^2+y^2=16$, $x^2=12(y-1)$

**2.** If $f(x)=\begin{cases} x\ , & for\ |x|\geq 1 \\ -x\ , & for\ |x|<1 \end{cases}$ and $g(x)=\frac{1}{2}\left|x^2-1\right|$, show that $\int_{-2}^{3} f(x)dx=g(3)-g(-2)=\frac{5}{2}$

**3.** If $f(x)=\begin{cases} \frac{1}{2}x^2\ , & for\ x\geq 0 \\ -\frac{1}{2}x^2\ , & for\ x<0 \end{cases}$ and $g(x)=\frac{1}{2}\left|x^2-1\right|$, show that $\int_{a}^{b}|x|dx=g(b)-g(a)$

**4.** Let $f:[a,b]\to\mathbb{R}$ and let $c\in\mathbb{R}$.

**(a)** If $F:[a,b]\to\mathbb{R}$ is an antiderivative of $f$ on $[a,b]$, show that $F_c(x)=F(x)+c$ is also an antiderivative of $f$ on $[a,b]$.

**(b)** If $F_1$ and $F_2$ are antiderivatives of $f$ on $[a,b]$, show that $F_1-F_2$ is a constant function on $[a,b]$.

**5.** Find the arc length of the given curves:

**(a)** $y=\ln(\sec x)$, lying between $x=0$ and $x=\frac{\pi}{3}$ **(b)** $y=\frac{1}{4}\ln x$, from $x=1$ to $x=e$

**(c)** $y=\arcsin\left(e^{-x}\right)$, from $x=0$ to $x=1$ **(d)** $y=\ln x$, from $x=1$ and $x=e$

**6.** Find a second degree polynomial $P(x)$ such that $P(0)=P(1)=0$ and $\int_{0}^{1}P(x)dx=1.$

**7.** Show that $\int_a^b f(x)\,dx = (b-a)\int_0^1 f\left(a+(b-a)x\right)dx$

**8.** Show that $\int_a^b f(x)\,dx = \frac{1}{\lambda}\int_{\lambda a}^{\lambda b} f\left(\frac{x}{\lambda}\right)dx, \quad (\lambda \neq 0)$

**9.** Let $f$ be a continuous and odd function. Show that $\int_0^{\pi} f(\cos x)\,dx = 0.$

**10.** Compute the integrals: $\int_0^{\pi/2} \frac{a\cos^3 x + b\sin^3 x}{\cos x + \sin x}\,dx = ?$ and $\int_0^{\pi/2} \frac{b\cos^3 x + a\sin^3 x}{\cos x + \sin x}\,dx = ?$

**11.** Let $f:[0,1] \to \mathbb{R}$ be a continuous function and $f(x) > 0$ for all $x \in [0,1]$.

Evaluate: $\int_0^1 \frac{f(x)}{f(x)+f(1-x)}\,dx = ?$

**12.** Compute the integrals: $\int_0^{\pi} |\cos x - \sin x|\,dx = ?$ and $\int_0^{\pi} |\cos x + \sin x|\,dx = ?$

**13.** Compute the integral: $\int_{-2}^{\frac{-1}{2}} \sqrt{\frac{2+x}{1-x}}\,dx = ?$ $\left(\textit{Hint. Do substitution } x = 1-\cos^2 u\right)$

**14. (a)** Show that: $\int_0^1 x^m\left(1-x^2\right)^n dx = \int_0^{\pi/2} \sin^m u.\cos^{2n+1} u\,du$ $\left(\textit{Hint. Do substitution } x = \sin u\right)$

**(b)** By using (a) compute: $\int_0^1 x^3\left(1-x^2\right)^{10} dx = ?$

**15.** Let $f$ and $g$ be continuous functions on $[a,b]$ and have second derivatives on $[a,b]$.

If $f(a) = g(a) = f(b) = g(b) = 0$, then show that $\int_a^b f(x)g''(x)\,dx = \int_a^b f''(x)g(x)\,dx$.

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